



# Robot Control for Magnetic Microswimmers

Nicolas Andreff

Institut FEMTO-ST

Univ. Bourgogne Franche-Comté / CNRS

Besançon, France

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Baptiste Véron (UFC 2014), Tiantian Xu (UPMC 2014),  
Ali Oulmas (UPMC 2018) and Maxime Etiévant (UBFC 2021)

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Fréjus, Oct. 28, 2021

# What is a robot ?



*In fact, the term “robot” means different things to different people.*

*Even roboticists themselves have different notions about what is or isn't a robot.*

*And for most of us, science fiction has strongly influenced what we expect a robot to look like and be able to do.*

*So what makes a robot? Here's a definition that is neither too general nor too specific:*

***A robot is an autonomous machine capable of sensing its environment, carrying out computations to make decisions, and performing actions in the real world.***

<https://robots.ieee.org/learn/what-is-a-robot/>

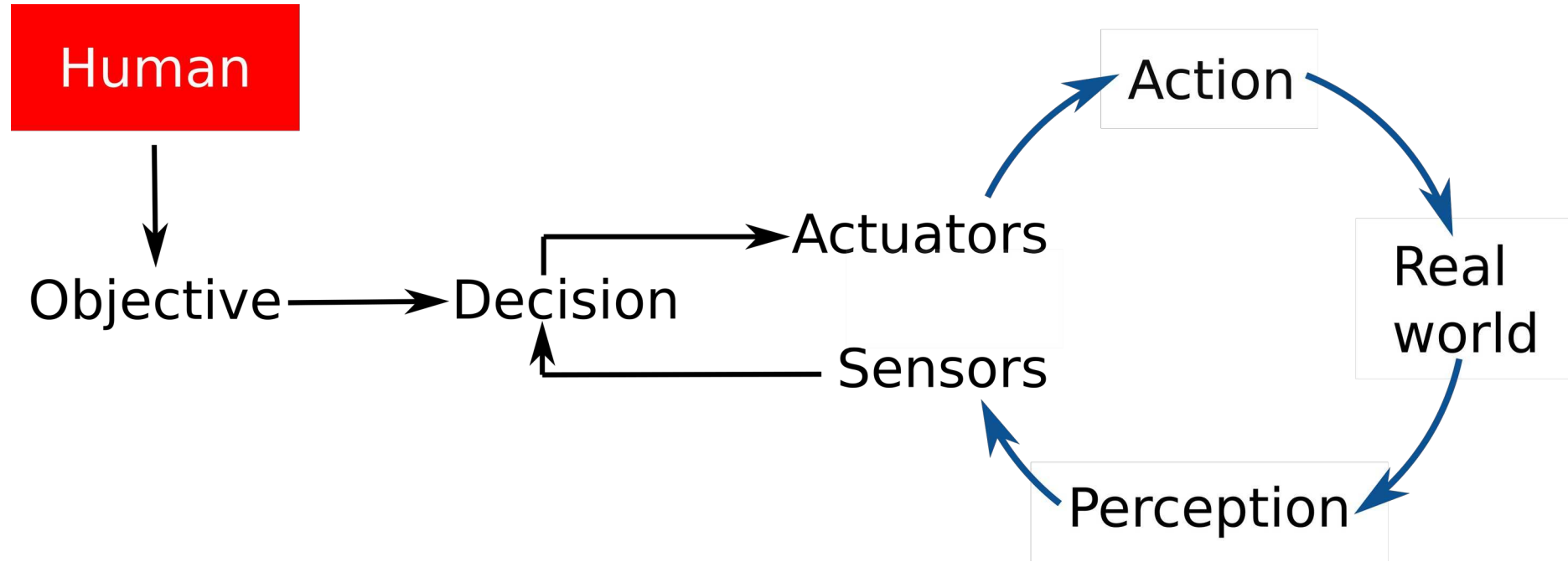
**Beware of fuzzy and/or human-related vocabulary (cf. magic AI) !**

**A robot is an autonomous machine capable of sensing its environment, carrying out programmed computations, and performing motion in the real world to achieve human-specified objectives.**

# What is a robot ?

The perception/action cycle

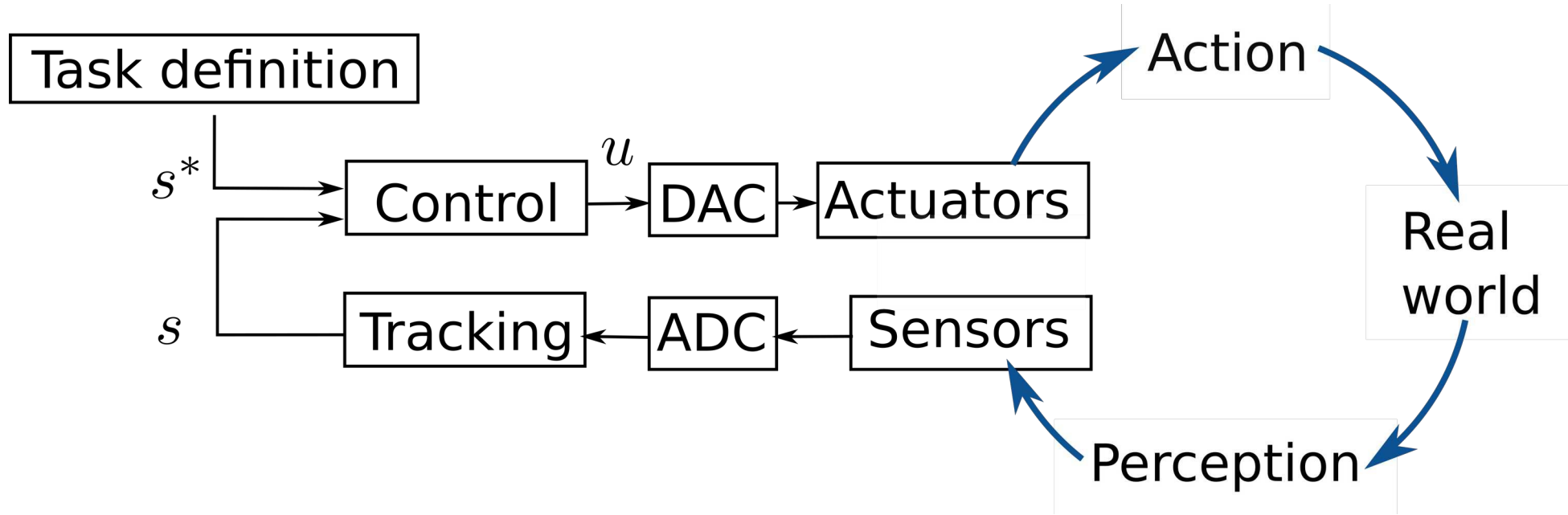
with a human-biased description of the robot structure



# What is a robot ?

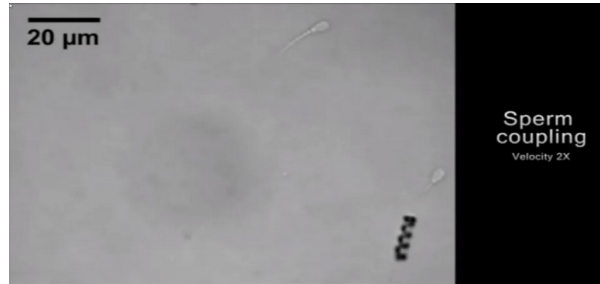
## The perception/action cycle

seen from the robot control standpoint

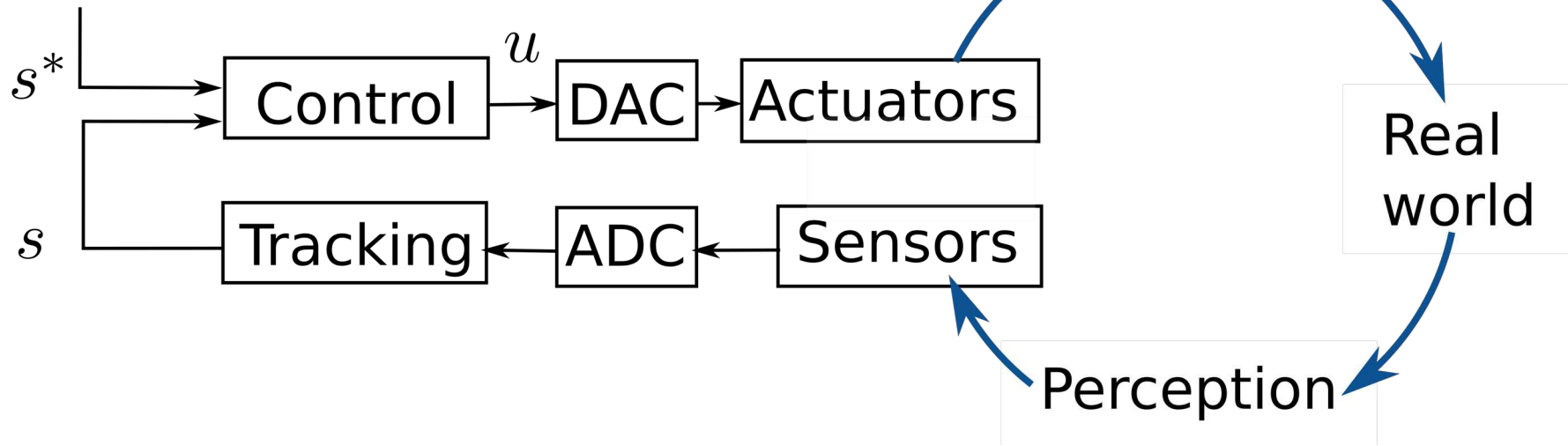


# Is a magnetic microswimmer a robot ?

[Medina-Sanchez14]



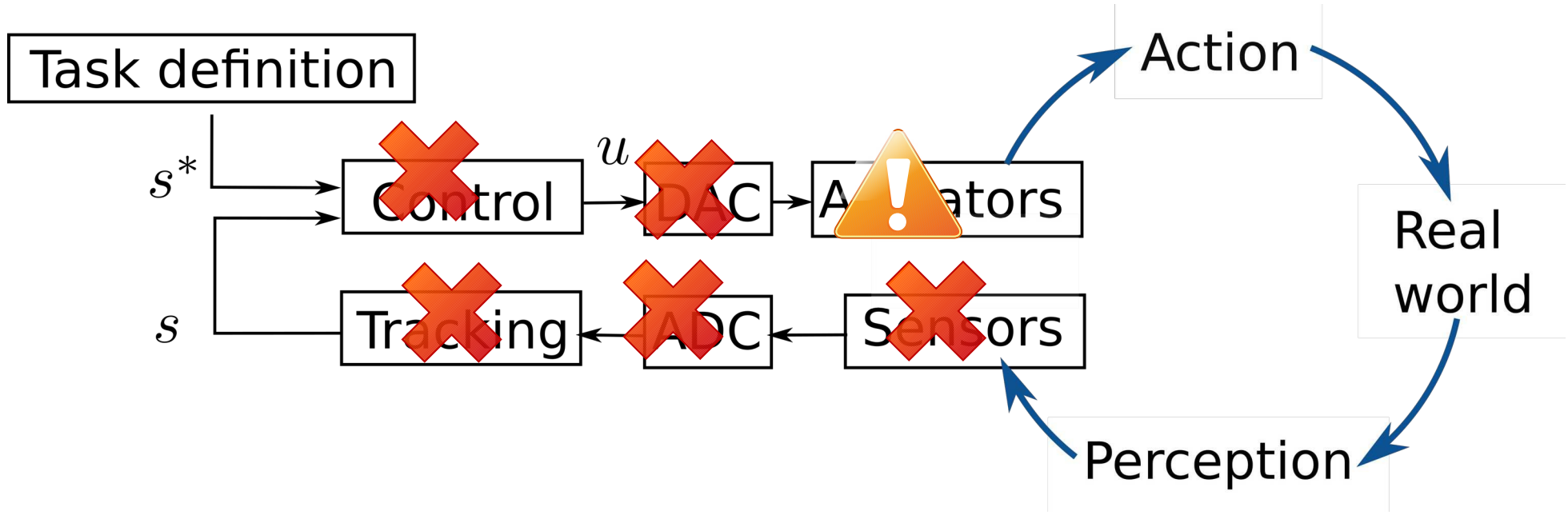
Task definition



# Is a magnetic microswimmer a robot ?

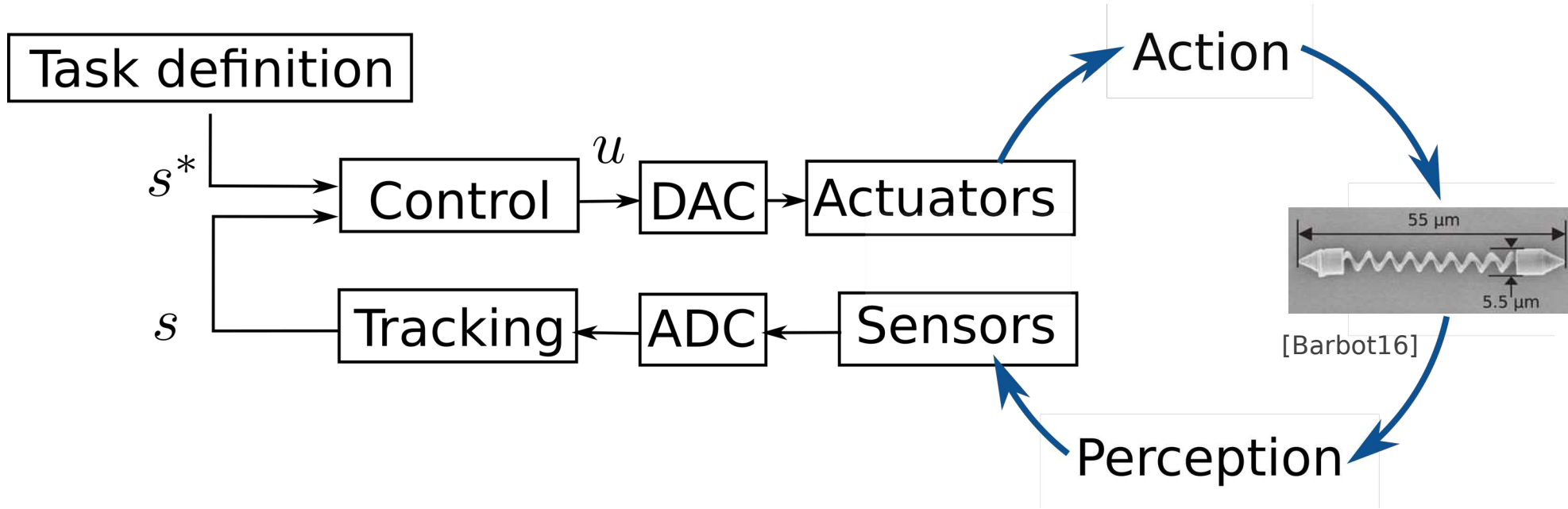


No



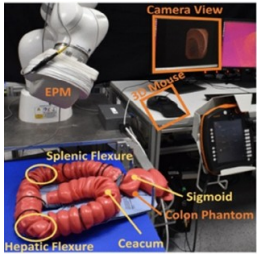
# Is a magnetic microswimmer a robot ?

No, but a magnetic manipulation system is !

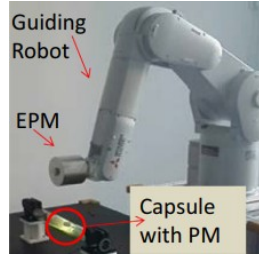




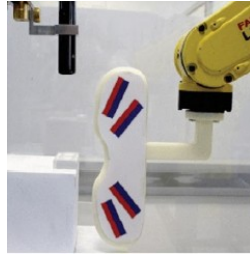
# A short tour of magnetic manipulation systems



[Pittiglio19]



[Li18]



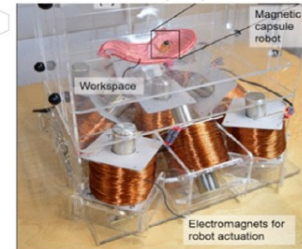
[Zarrouk19]



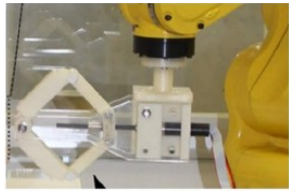
[Siemens/Olympus 12]



[Diller14]



[Son19]



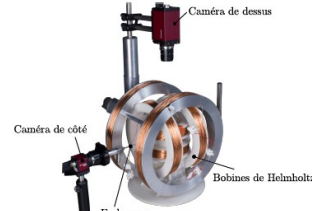
[Amokrane18]



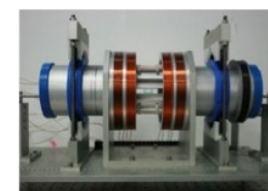
[Liao16]



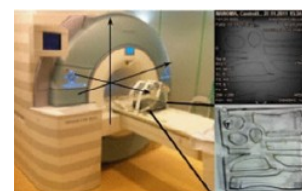
[Armacost07]



[Oulmas17]



[Choi10]



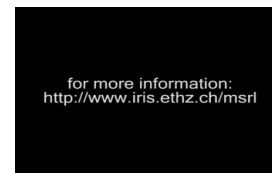
[Folio17]



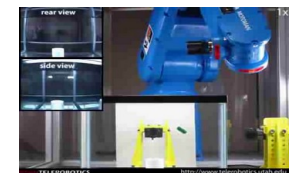
[Ciutti 10]



[Stereotaxis 10]



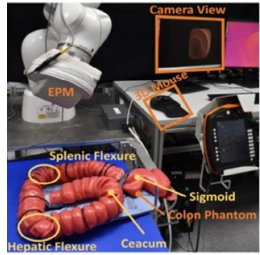
[Kummer 10]



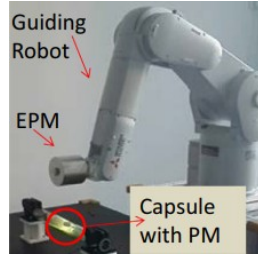
[Mahoney 11]



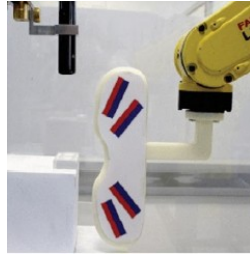
# A short tour of magnetic manipulation systems



[Pittiglio19]



[Li18]



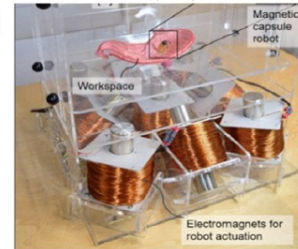
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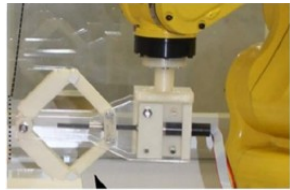
[Siemens/Olympus 12]



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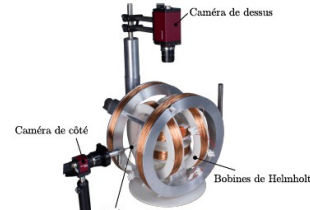
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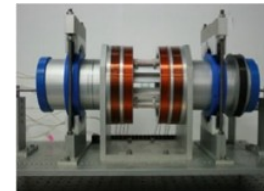
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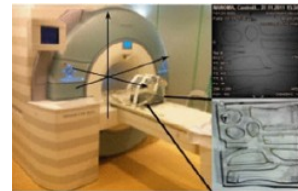
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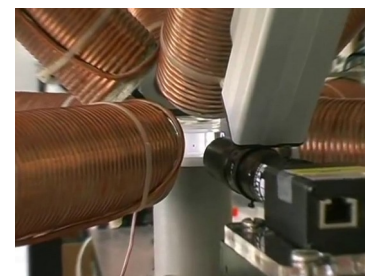
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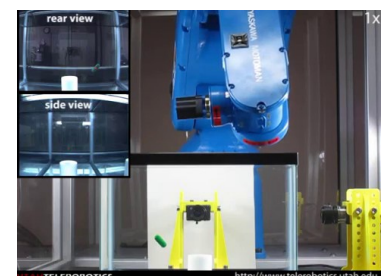
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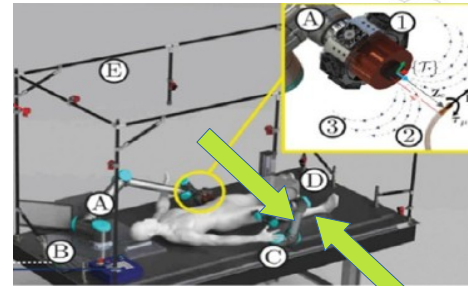
# Various kinds of magnetic manipulation systems



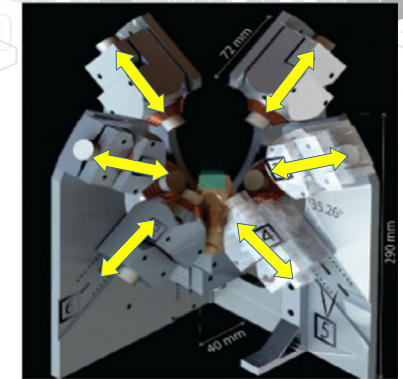
- **Mobile permanent magnets**
- **Static electromagnets**
- **Mobile electromagnets** [Véron 12]

# Various kinds of magnetic manipulation systems

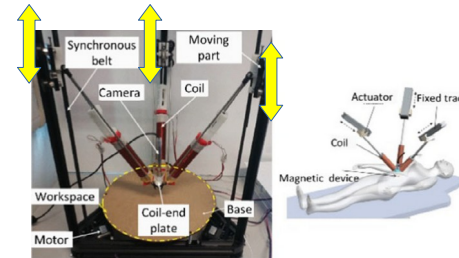
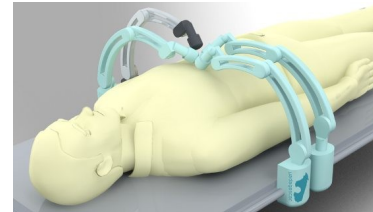
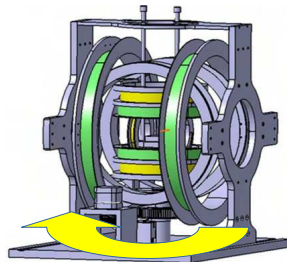
- **Mobile permanent magnets**
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- **Mobile electromagnets** [Véron 12]



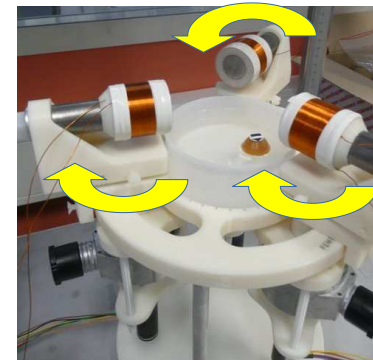
[Sikorski 19]



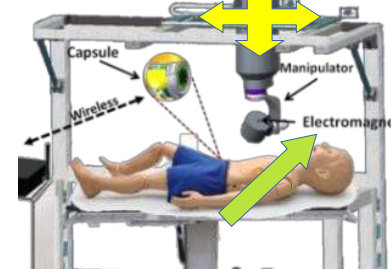
[Ongaro 19]



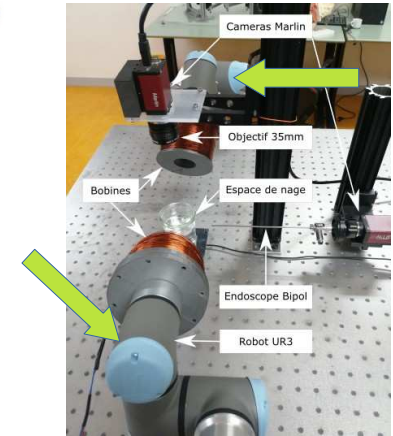
[Yang 19]



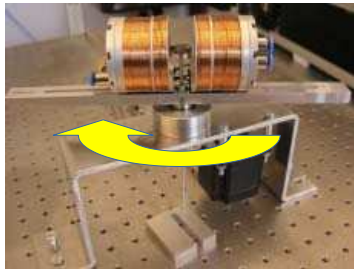
[Véron 12]



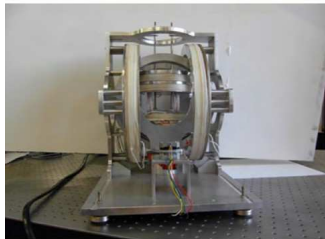
[Lucarini 15]



[Etiévant 21]



[Yesin 06]



[Yu 10]

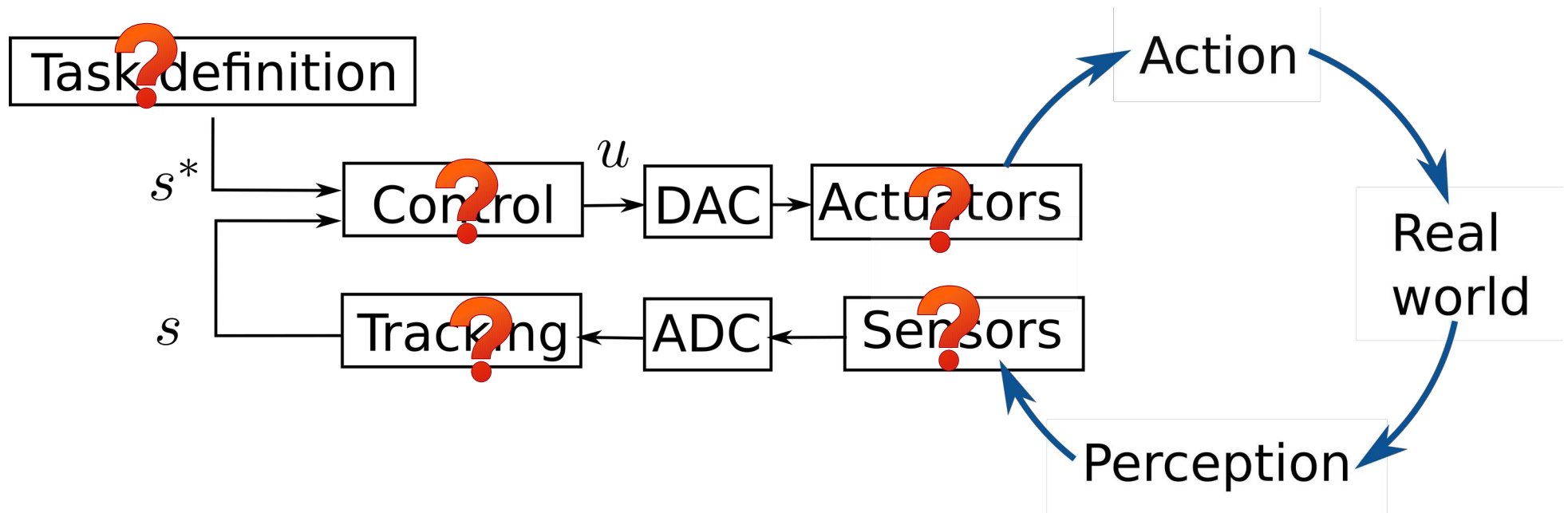
# Pros and cons



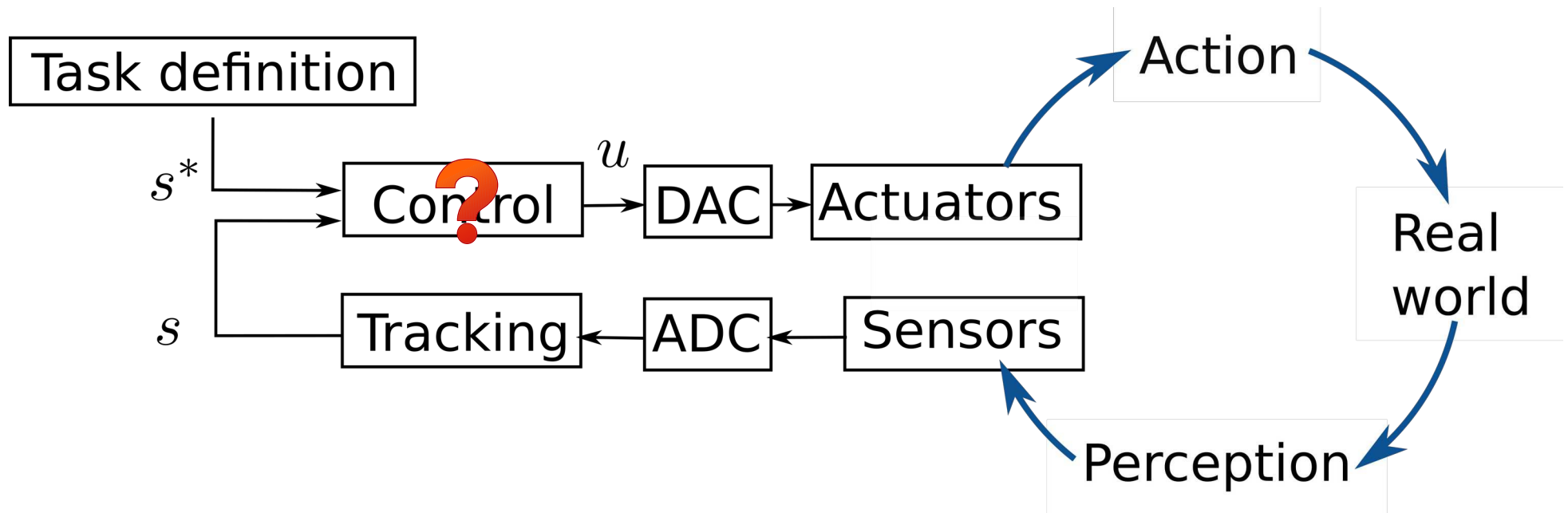
Property	Static electromagnets	Mobile permanent magnets	Mobile electromagnets
Dexterity	⊕ complete	⊖ partial	⊕ complete
Source/object distance	⊖ long	⊕ short	⊕ short
Joule effect	⊖ strong	⊕ inert	↔ reduced
Turn off the field	⊕ possible	⊖ impossible	⊕ possible
Stabilization	⊕ possible	⊖ unstable/difficult	⊕ possible
Control mode	⊕ simple	⊕ simple	⊕ redundant
Patient acceptance	⊖ weak	⊖ dangerous	↔ improved



# What is a magnetic manipulation system?



# What is a magnetic manipulation system?



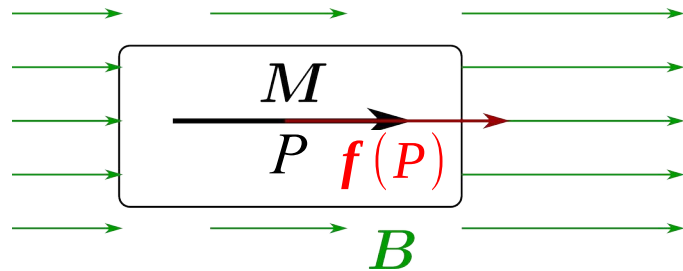
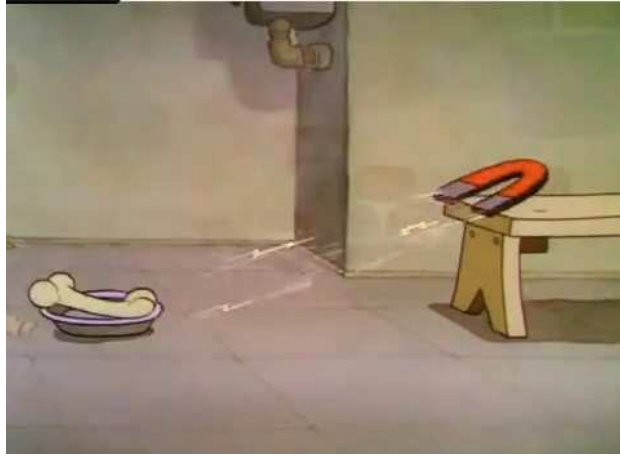
# Control issues

- **Robot structure**
- Robot model
- Kinematic analysis
- Desired motion
- Control scheme

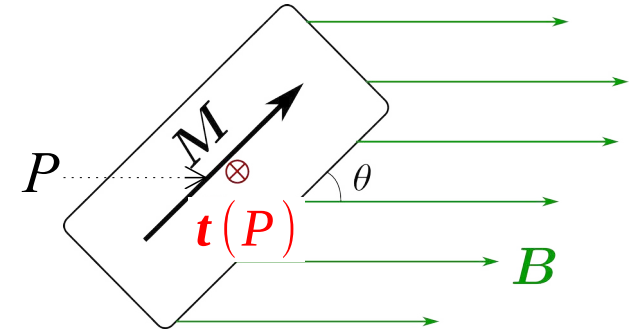




# Magnetic efforts on a magnetic object



$$f(P) = V \cdot \nabla (M \cdot B(P))$$



$$t(P) = V \cdot M \wedge B(P)$$

# Several actuation modes



- **Force and Torque (F/T)**
  - Dynamic equations
- **Force and Field (F/B)**
  - Higher dynamics in rotation than in translation :  $M$  (almost) always aligned with  $B$ , thus  $T=0$
- **Field only (B)**
  - Uniform field  $\rightarrow F = 0$
  - Uniform field easy to produce : Helmholtz configuration
  - Force scales down poorly
- **Force only (F)**
  - Less frequent because  $B \neq 0$
  - Bead pulling

# Control issues

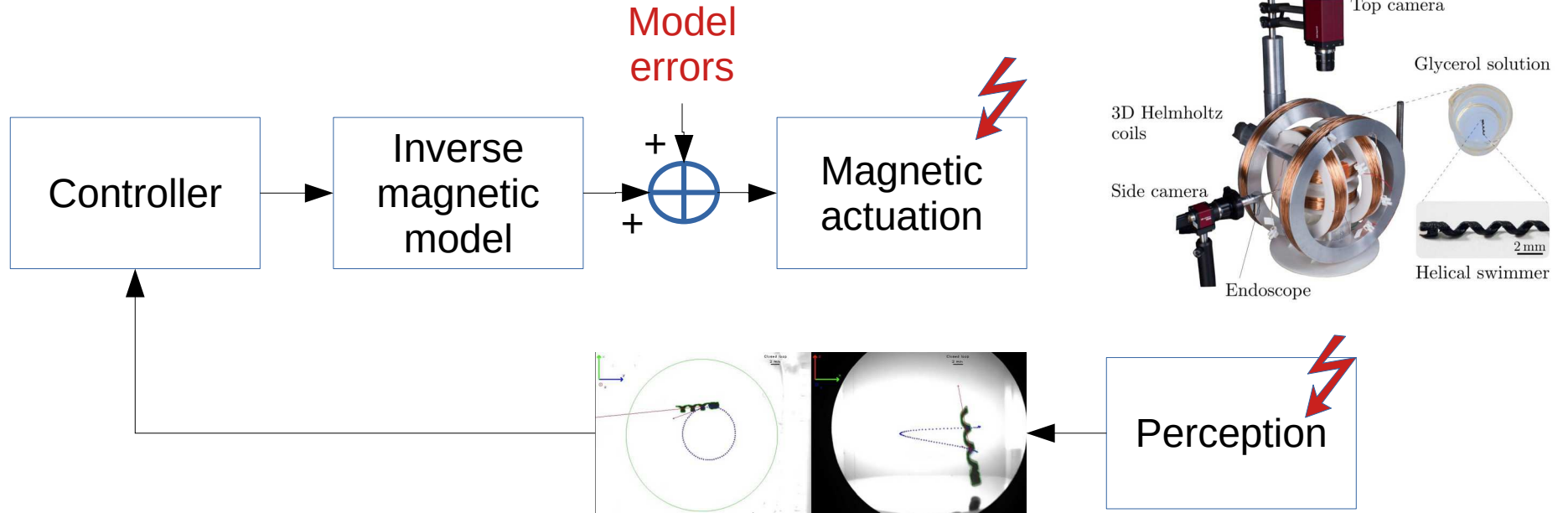
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# Magnetic field model

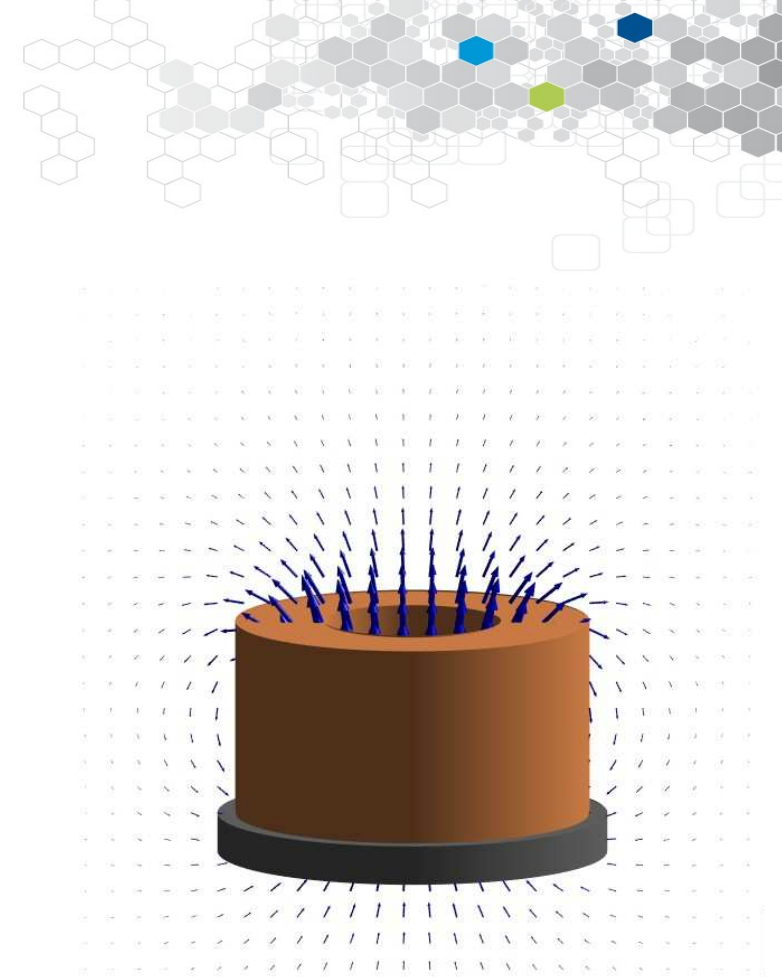
- **Requirements**

- Fast → closed-loop control, real-time constraint
- Accurate ... but sensor-based control, so not *that* accurate



# Magnetic field models

- **Specific configurations**
  - Helmholtz pair :  $B = \text{cst}$  at center ( $\rightarrow B \approx \text{cst}$  everywhere)
  - Maxwell pair :  $B = 0$  at center ( $\rightarrow F \approx \text{cst}$  everywhere)
- **General case**
  - $\mathbf{B}(P) = \mathbf{b}(P) \cdot I$
  - Fast decay of  $B$  as  $P$  goes away
  - Non linear close to the source, smooth (linear) far away
  - Complex

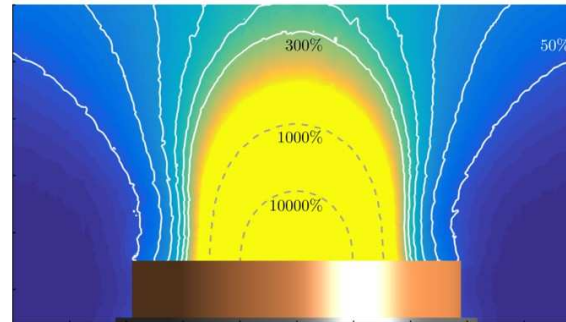


Champ magnétique produit par une bobine

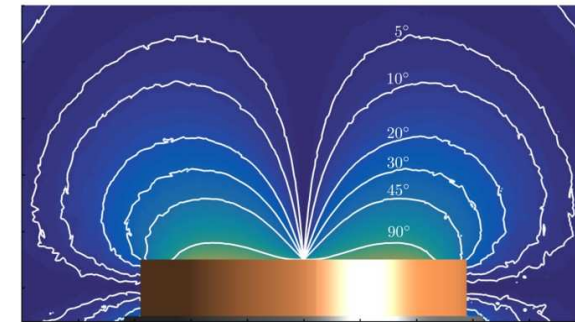
# Magnetic field models



- **Finite Element Model**
  - Accurate but sparse
  - Costly but pre-computed
  - Interpolation between samples but memory access cost
- **Mapping**
  - Same as above
  - Fits for any shape
- **Dipole model**
  - Circular loop
  - $r \gg a$
  - Simple closed-form expression



Relative error on field norm  
wrt. FEM



Angular error  
wrt. FEM

# Models based on magnetic potential vector

- **Circular loop**

$$\mathbf{b} = \nabla \wedge \mathbf{a} \quad \text{with} \quad \mathbf{a} = (A_r, A_\theta, A_\phi)^T \quad [\text{Jackson99}]$$

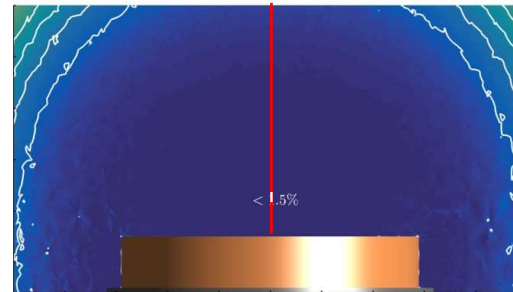
$$\begin{cases} A_r = 0 \\ A_\theta = 0 \\ A_\phi(r, \theta) = \frac{\mu_0 i}{\pi} \frac{a}{\sqrt{a^2 + 2ar \sin \theta + r^2}} \left[ \frac{(2 - k^2) K(k) - 2E(k)}{k^2} \right] \\ k(r, \theta) = \sqrt{\frac{4ar \sin \theta}{a^2 + 2ar \sin \theta + r^2}} \end{cases}$$

Elliptic integral functions

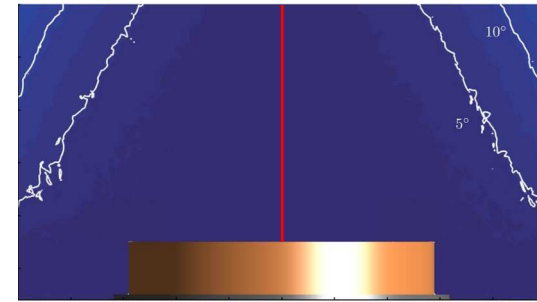
$$K(k) = \int_0^{\frac{\pi}{2}} \frac{1}{\sqrt{(1 - k^2 \sin^2 \alpha)}} d\alpha$$

$$E(k) = \int_0^{\frac{\pi}{2}} \sqrt{(1 - k^2 \sin^2 \alpha)} d\alpha$$

- Almost as accurate as FEM
- Higher CPU cost : derivatives of K & E
- Not defined for  $r = 0$  or  $\theta = 0$



Relative error on field norm  
wrt. FEM



Angular error  
wrt. FEM



# Control-oriented magnetic field model

- Model based on the magnetic potential vector
- Use of an “old” formula

$$\frac{dK}{dk}(k) = \frac{E(k)}{k(1-k^2)} - \frac{K(k)}{k}$$

$$\frac{dE}{dk}(k) = \frac{E(k)}{k} - \frac{K(k)}{k}$$

[Abramowitz72]



Pour tout  $(r, \theta) \in D_{k^*}$  on a :

$$\begin{cases} b_r(r, \theta) = \frac{\mu_0 I}{\pi} \frac{a^2}{\sqrt{a^2 + 2ar \sin \theta + r^2}} \frac{E(k) \cos \theta}{a^2 - 2ar \sin \theta + r^2} \\ b_\theta(r, \theta) = \frac{\mu_0 I}{\pi} \frac{1}{\sqrt{a^2 + 2ar \sin \theta + r^2}} \left[ \frac{E(k) (r^2 + a^2 \cos(2\theta))}{(a^2 - 2ar \sin \theta + r^2) 2 \sin \theta} \right] \\ b_\phi = 0 \end{cases}$$

[Etiévant 19]

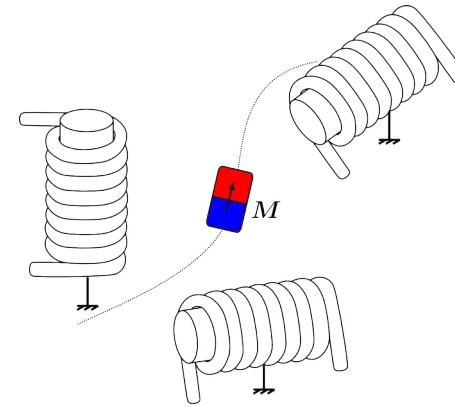
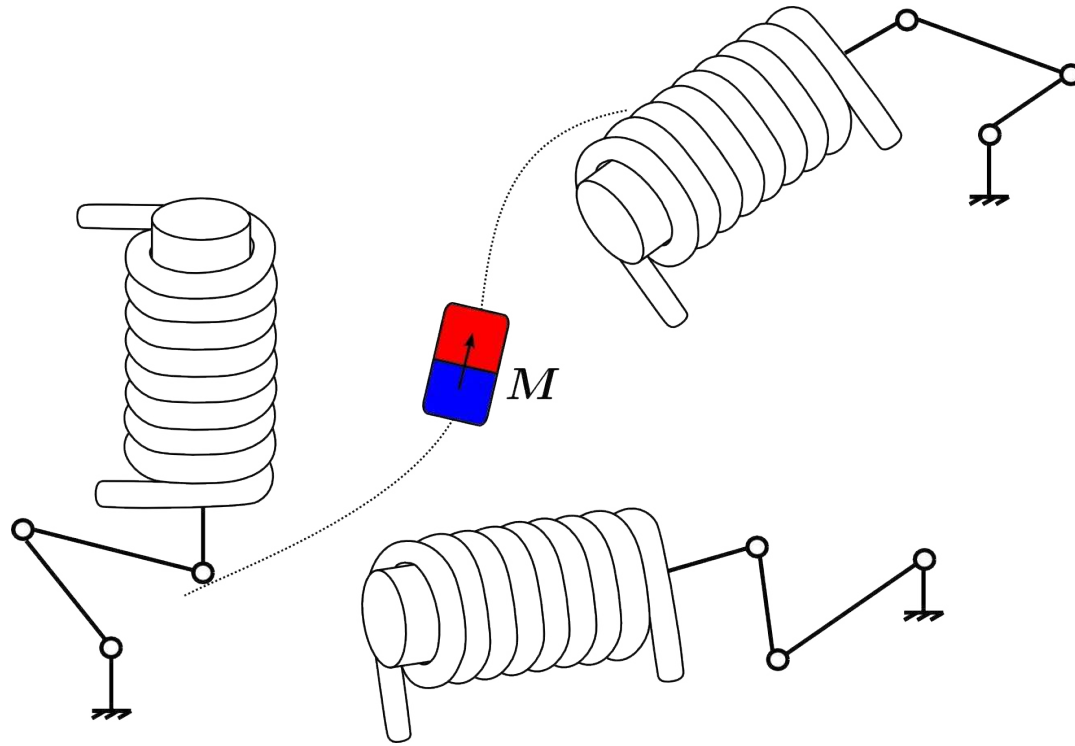
- Smooth extension on the axis

	Computation time (ms/point)	Standard deviation (ms)	Memory used (ko)
Mapping	162	1.583	$\geq 1400$
Dipole	1.7	0.163	$\leq 6$
Wong's formulation	1164.5	7.1	$\leq 7$
Schill's formulation	241.4	2.5	$\leq 7$
Extended formulation	3.6	0.147	$\leq 7$

# A generic model for multi-mobile source systems

Specific case

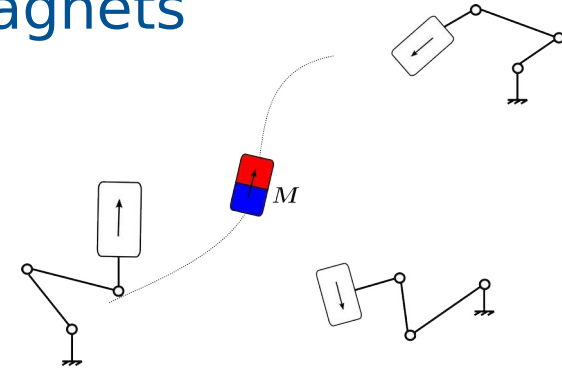
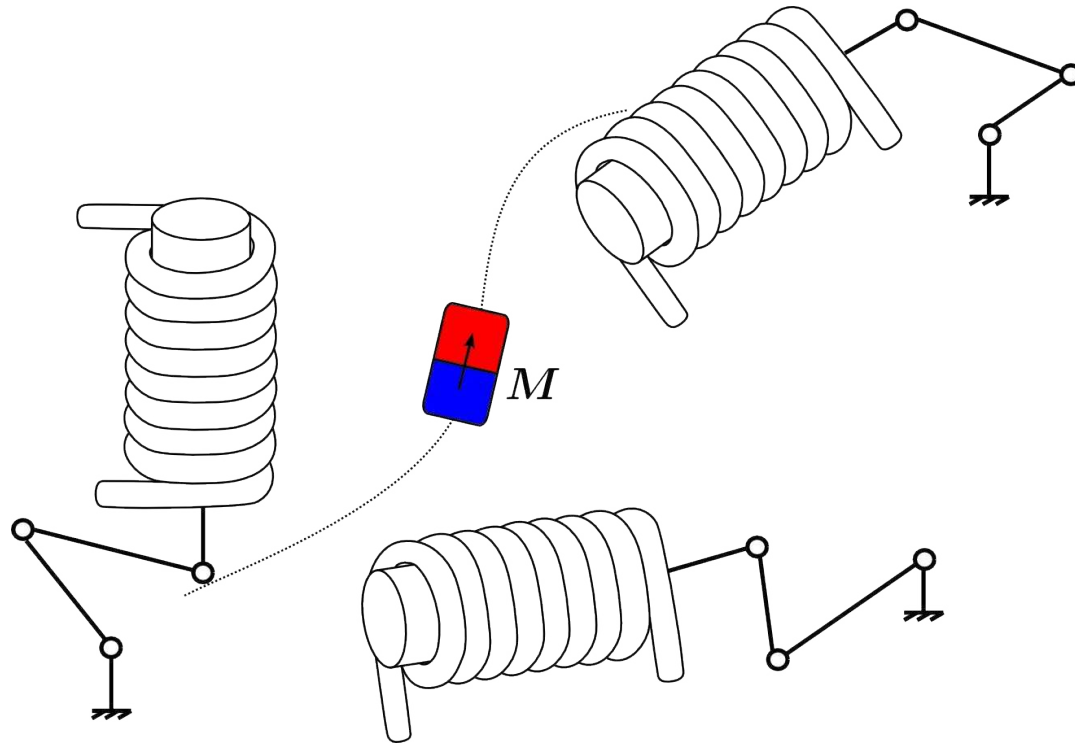
Static electromagnets



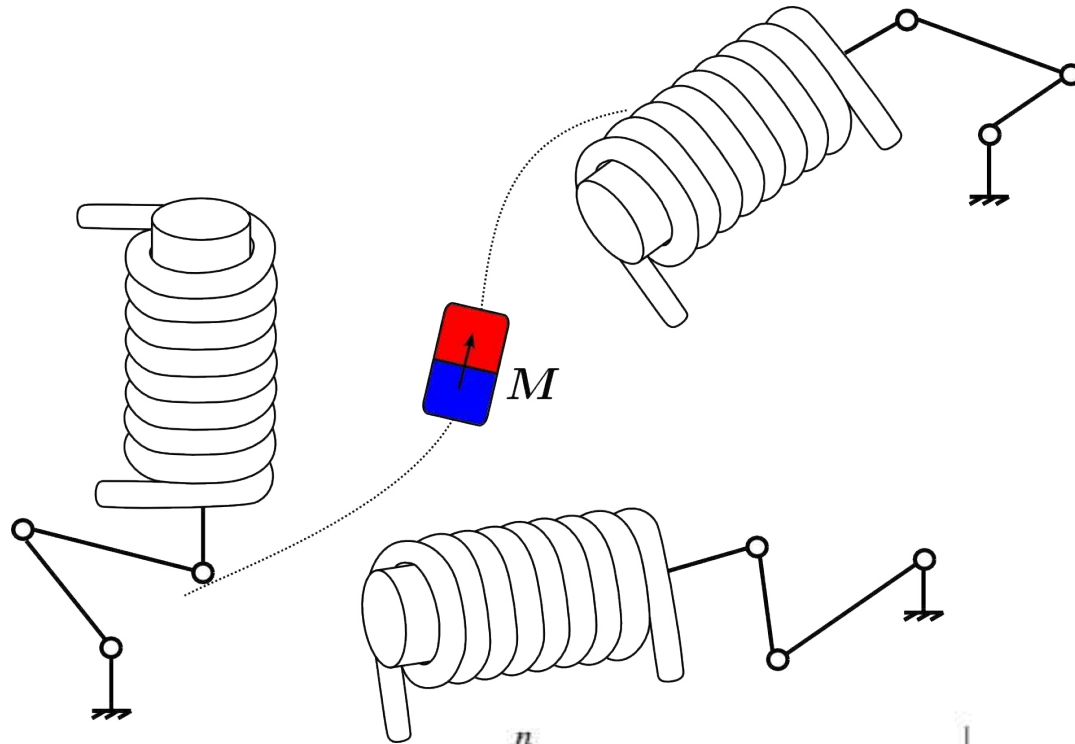
# A generic model for multi-mobile source systems

Specific case

Mobile permanent magnets



# A generic model for multi-mobile source systems



- Superposition theorem
- Linear in the currents
- Non-linear in the source locations
- In robotic language:  
**Forward Electromagnetic Model (FEmM)**

$$\begin{bmatrix} \mathbf{f} \\ \mathbf{b} \end{bmatrix} = FEmM(\{\mathbf{}^0T_i\}_{i=1..n}, \mathbf{I})$$

$$\begin{aligned} \mathbf{B}(\{\mathbf{}^jT_0\}_j, \mathbf{}^0\tilde{\mathbf{p}}) &= \sum_{j=1}^n \mathbf{}^0R_j \mathbf{}^j\mathbf{b}_j(\mathbf{}^jT_0 \mathbf{}^0\tilde{\mathbf{p}}) i_j \\ &= \mathbf{A}(\{\mathbf{}^jT_0\}_j, \mathbf{}^0\tilde{\mathbf{p}}) \mathbf{i} \end{aligned}$$

$$\begin{aligned} \mathbf{f}(\{\mathbf{}^jT_0\}_j, \mathbf{}^0\tilde{\mathbf{p}}, \mathbf{m}) &= \sum_{j=1}^n \mathbf{}^0R_j \mathbf{}^j\mathbf{f}_j(\mathbf{}^jT_0 \mathbf{}^0\tilde{\mathbf{p}}, \mathbf{m}) i_j \\ &= \mathbf{F}(\{\mathbf{}^jT_0\}_j, \mathbf{}^0\tilde{\mathbf{p}}, \mathbf{m}) \mathbf{i} \end{aligned}$$

[Véron 14]

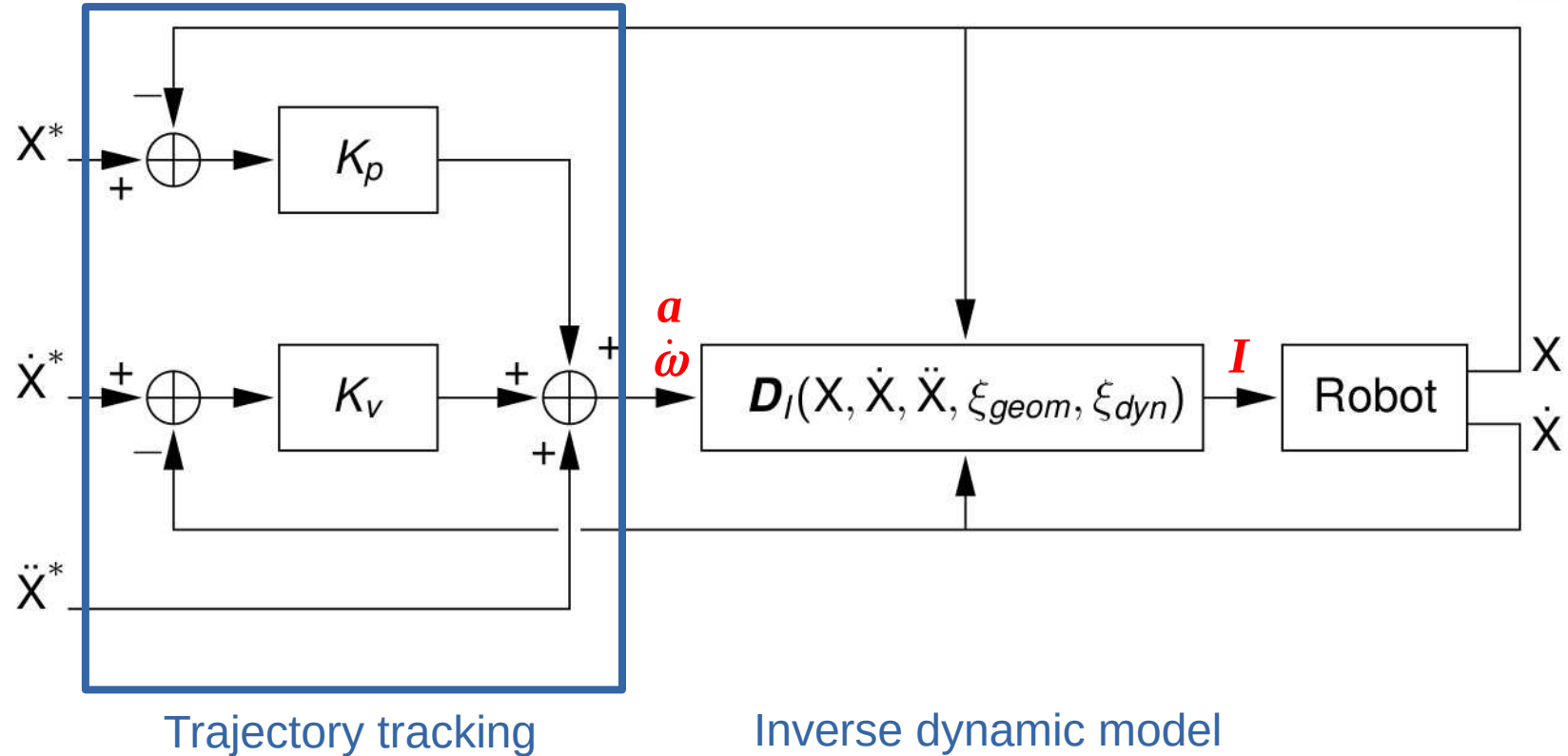
# Control issues

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- Desired motion
- **Control scheme**



# Dynamic control

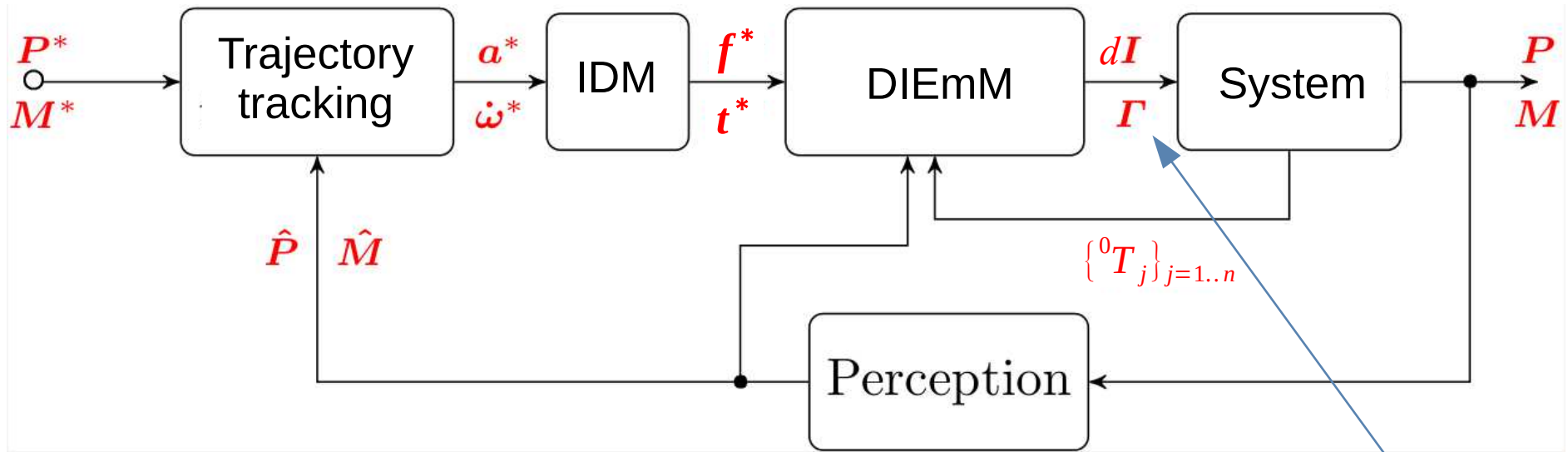
- Computed-torque control



# Computed Twist-and-Current Control



DIEmM =  
Differential Inverse Electromagnetic Model



$$\Gamma = \begin{bmatrix} \gamma_1 \\ \vdots \\ \gamma_n \end{bmatrix} \quad \gamma_j = \begin{bmatrix} \vec{v}_j \\ \vec{\Omega}_j \end{bmatrix}$$



# Non-linear control by linearisation



- **Forward Electromagnetic Model (FEmM)**

$$\begin{bmatrix} \mathbf{f} \\ \mathbf{t} \end{bmatrix} = \mathbf{F}_{EmM}(\{\{^0T_j\}_{j=1..n}, \mathbf{I}\})$$

non linear

- **Inverse Electromagnetic Model (IEmM)**

$$\langle \{\{^0T_j\}_{j=1..n}, \mathbf{I} \rangle = \mathbf{FEmM}^{-1}(\mathbf{f}, \mathbf{t})$$

solution to a non linear equation !  
→ Linearisation

- **Differential Forward Electromagnetic Model (DFEmM) / DIEmM**

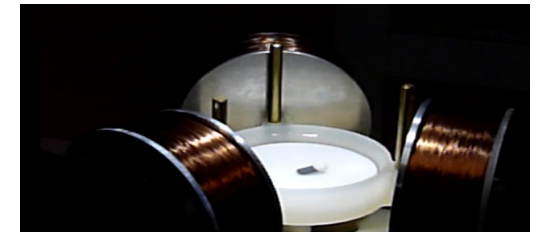
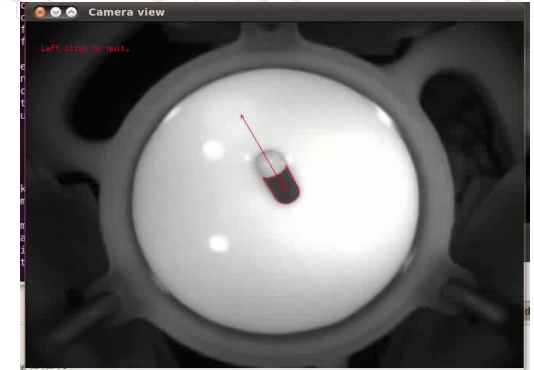
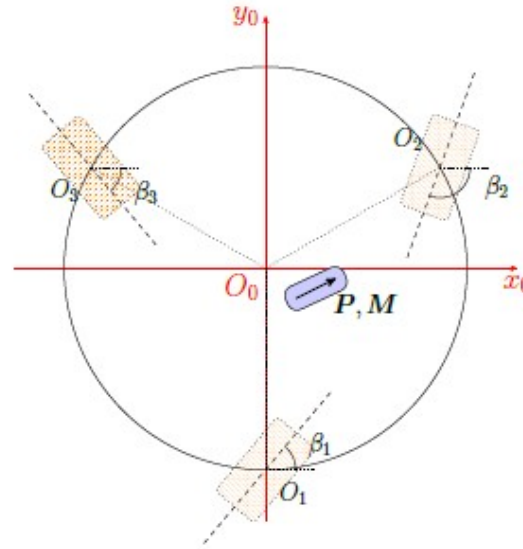
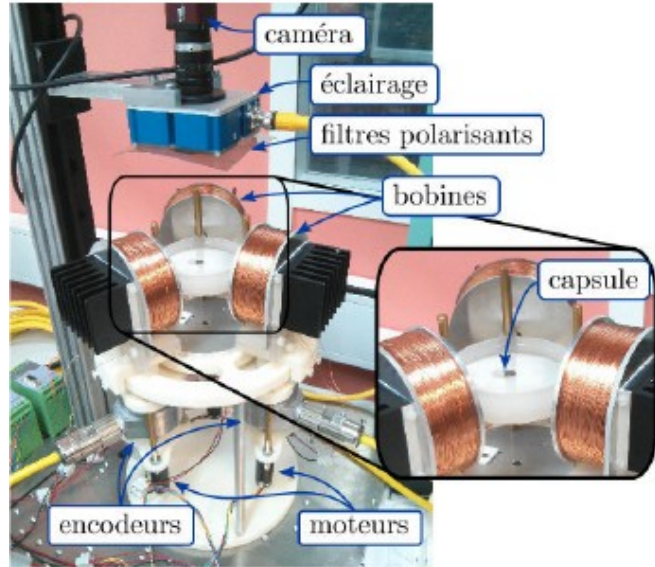
$$\begin{bmatrix} d\mathbf{f} \\ d\mathbf{t} \end{bmatrix} = \mathbf{J}_{Em}(\{\{^0T_j\}_{j=1..n}, \mathbf{I}\}) * \begin{bmatrix} d\mathbf{I} \\ \mathbf{\Gamma} \end{bmatrix}$$

$$\begin{bmatrix} d\mathbf{I} \\ \mathbf{\Gamma} \end{bmatrix} = \mathbf{J}_{Em}^{inv}(\mathbf{f}, \mathbf{t}) * \begin{bmatrix} d\mathbf{f} \\ d\mathbf{t} \end{bmatrix}$$

- **Proportional linearised controller**

$$\begin{bmatrix} d\mathbf{I} \\ \mathbf{\Gamma} \end{bmatrix} = k_p * \mathbf{J}_{Em}^{inv}(\mathbf{FEmM}(\{\{^0T_j\}_{j=1..n, k-1}, \mathbf{I}_{k-1}\})) * \begin{bmatrix} \mathbf{f}^* - \mathbf{f}_{k-1} \\ \mathbf{t}^* - \mathbf{t}_{k-1} \end{bmatrix}$$

# Experimental validation



# Control issues

- Robot structure
- Robot model
- **Kinematic analysis**
- Desired motion
- Control scheme





# « Kinematic » analysis

- **Singularities**

$$\begin{bmatrix} d\mathbf{f} \\ d\mathbf{t} \end{bmatrix} = \mathbf{J}_{Em}(\dots) * \begin{bmatrix} d\mathbf{I} \\ \mathbf{\Gamma} \end{bmatrix}, \quad \begin{bmatrix} d\mathbf{I} \\ \mathbf{\Gamma} \end{bmatrix} = \mathbf{J}_{Em}^{inv}(\dots) * \begin{bmatrix} d\mathbf{f} \\ d\mathbf{t} \end{bmatrix}$$

$$rank(\mathbf{J}_{Em}) dim(\mathbf{J}_{Em}) = 6 \times 7n \Rightarrow rank(\mathbf{J}_{Em}) \leq 5$$

if ( rank < 5 )  $\mathbf{J}_{Em}$  is singular => serial singularity : can not update the full wrench

- **Null space**

$$dim(\mathbf{J}_{Em}) = 6 \times 7n \Rightarrow rank(\mathbf{J}_{Em}) < 5 \Rightarrow dim ker(\mathbf{J}_{Em}) \geq 7n - 5$$

Many infinitely many ways to change the configuration of the magnetic manipulator for the control of 1 magnetic object/tool

→ Manipulate several objects : max 5 dof/object, thus up to  $7n/5$  objects !

→ Use this redundancy to optimise additional cost functions

e.g. manipulability, collisions, energetic cost, ...



# « Kinematic » analysis

- Manipulability**

$$\begin{bmatrix} df \\ dt \end{bmatrix} = J_{Em}(\dots) * \begin{bmatrix} dI \\ \Gamma \end{bmatrix}$$

How “good” is  $J_{Em}$  at updating the wrench ?

Numerical index : condition number  $\kappa = \frac{\sigma_1}{\sigma_6}$

First manipulability index :  $\mu = \frac{1}{\kappa}$

Many other manipulability indices

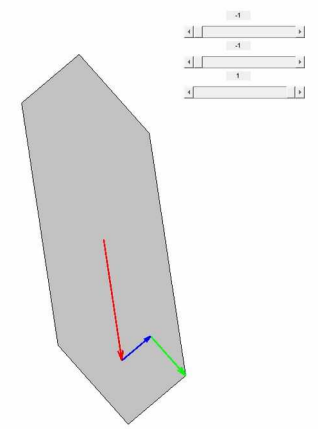
- From PKM wrench feasible workspace...**

$$WFW = \{ \mathbf{F} \in \mathbb{R}^6 \mid \mathbf{F} = \mathbf{J}^{-T} \cdot \boldsymbol{\tau}, 0 \leq \tau \leq \tau_{max} \}$$

Tension  
↓

**... to the achievable field workspace**

$$E_B(\{ {}^0T_j \}_{j=1..n}, P) = \left\{ \mathbf{B}(\{ {}^0T_j \}_{j=1..n}, P), \mathbf{B}(\dots) = \sum_{j=1}^n \mathbf{b}_j(\dots) I_j, |I_j| \leq I_{max} \right\}$$



# « Kinematic » analysis

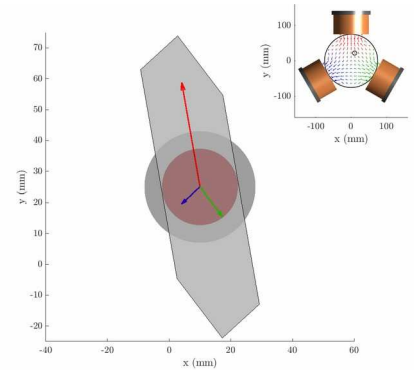
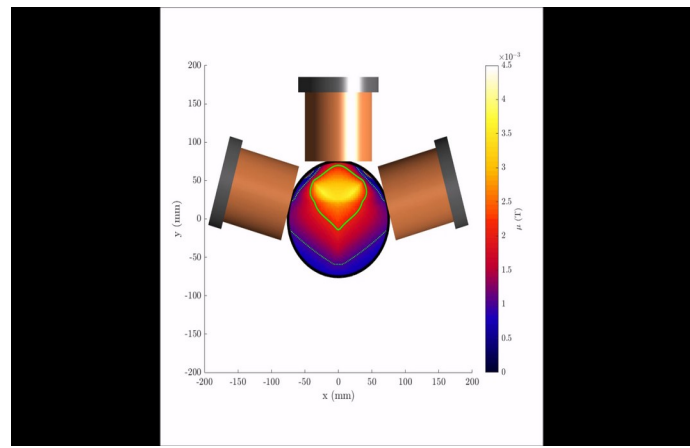
- From the achievable field workspace

$$E_B(\{^0T_j\}_{j=1..n}, P) = \left\{ \mathbf{B}(\{^0T_j\}_{j=1..n}, P), \mathbf{B}(\dots) = \sum_{j=1}^n \mathbf{b}_j(\dots) I_j, |I_j| \leq I_{max} \right\}$$

... to a magnetic manipulability index

$$\mu(P) = \max_{\vec{e}} \left\{ \mathbf{B}(\{^0T_j\}_{j=1..n}, P) \cdot \vec{e}, \mathbf{B}(\dots) \in E_B, |\vec{e}| = 1 \right\}$$

- Dexterity map



# Control issues

- Robot structure
- Robot model
- Kinematic analysis
- **Desired motion**
- Control scheme

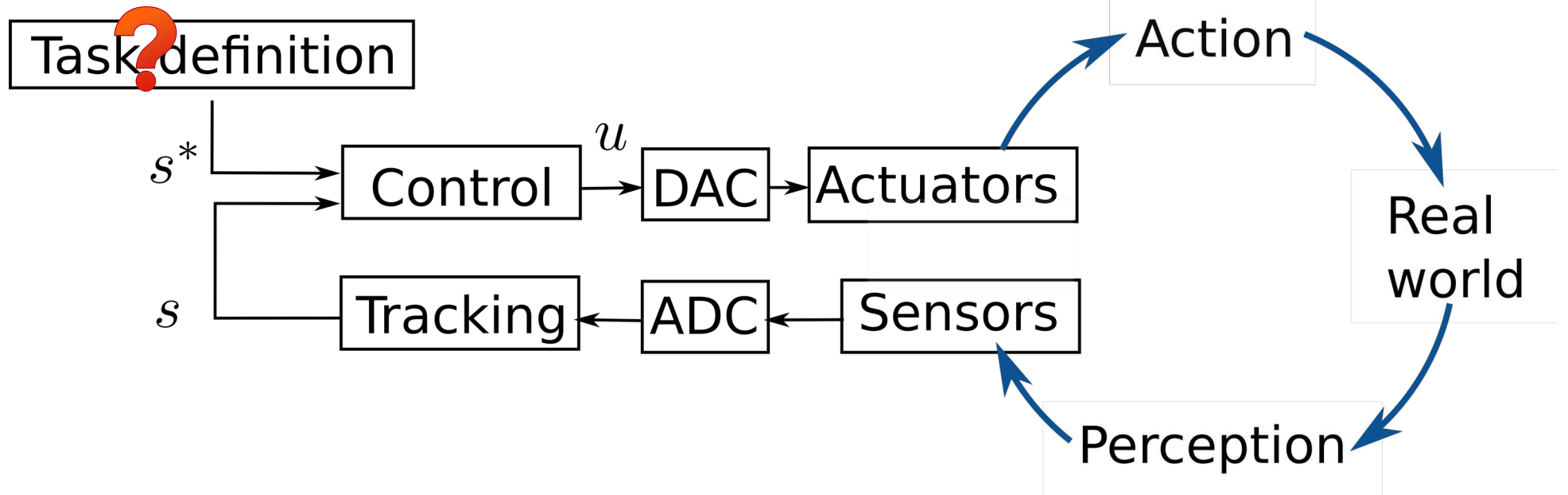




# Keep control at any time !



# What is a magnetic manipulation system?



# User needs and robotics practice



- **User ≠ robotics expert**
- **User needs and Task design**
  - Ergonomic
  - Simple
  - Accurate



- **Trajectory definition**

- Discrete set of waypoints

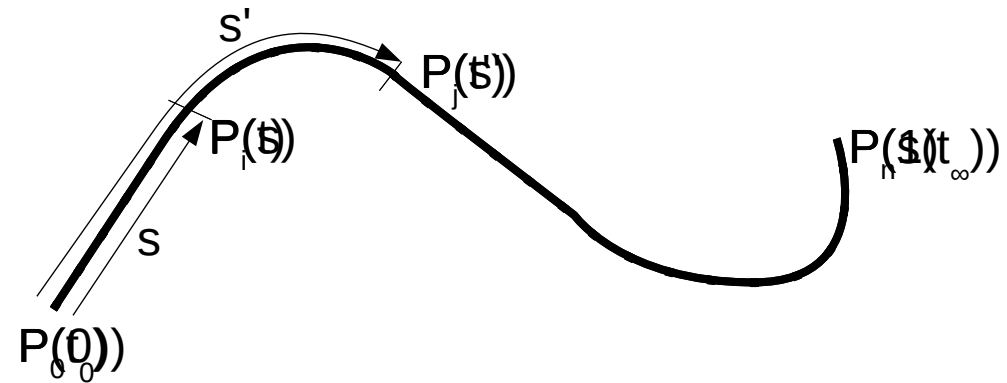
$$\mathbf{S} = \{ \mathbf{P}_i, i = 1..n \}$$

- Path = continuous curve

$$\mathbf{C} = \{ \mathbf{P}(s), s \in [0,1] \}$$

- Trajectory = timed waypoints

$$\mathbf{T} = \{ \mathbf{P}(t), t \in [t_0, t_{inf}] \}$$

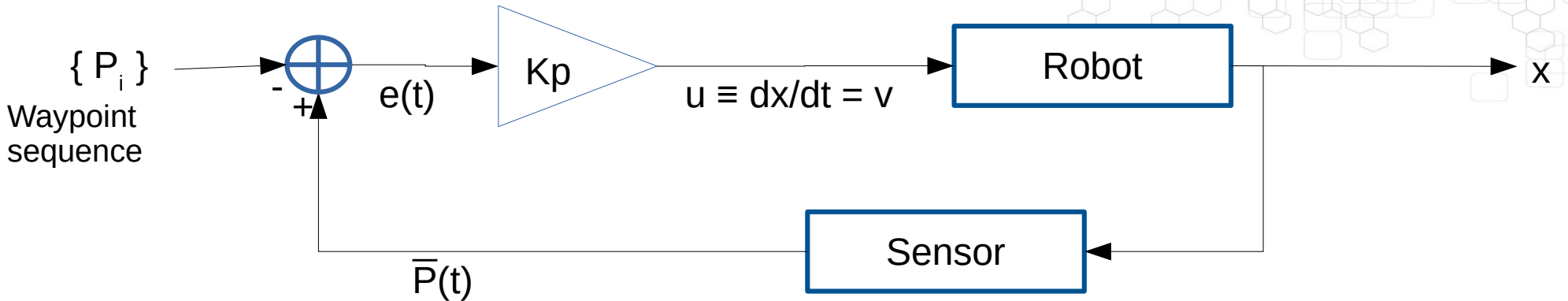


# Control issues

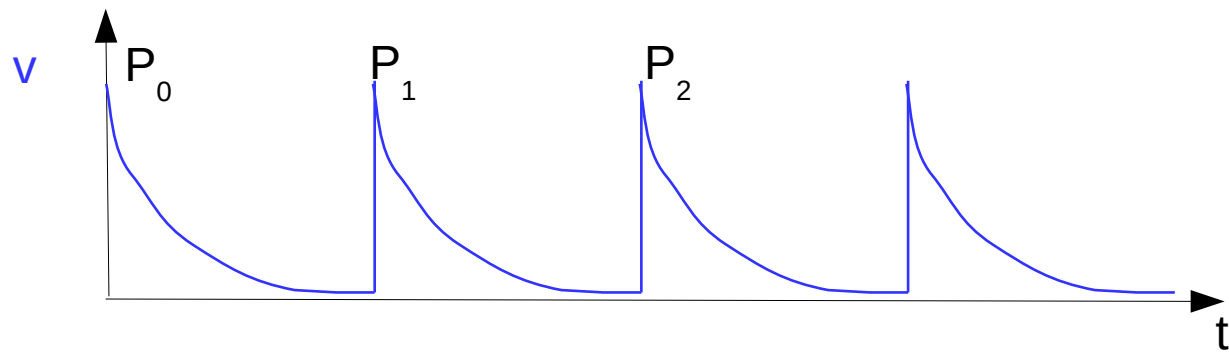
- Robot structure
- Robot model
- Kinematic analysis
- Desired motion
- **Control scheme**



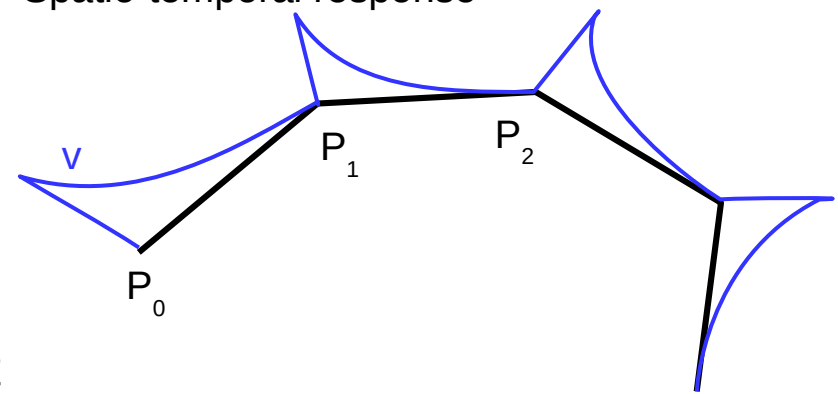
# Standard controller



Time response



Spatio-temporal response



# Standard controller at small scale

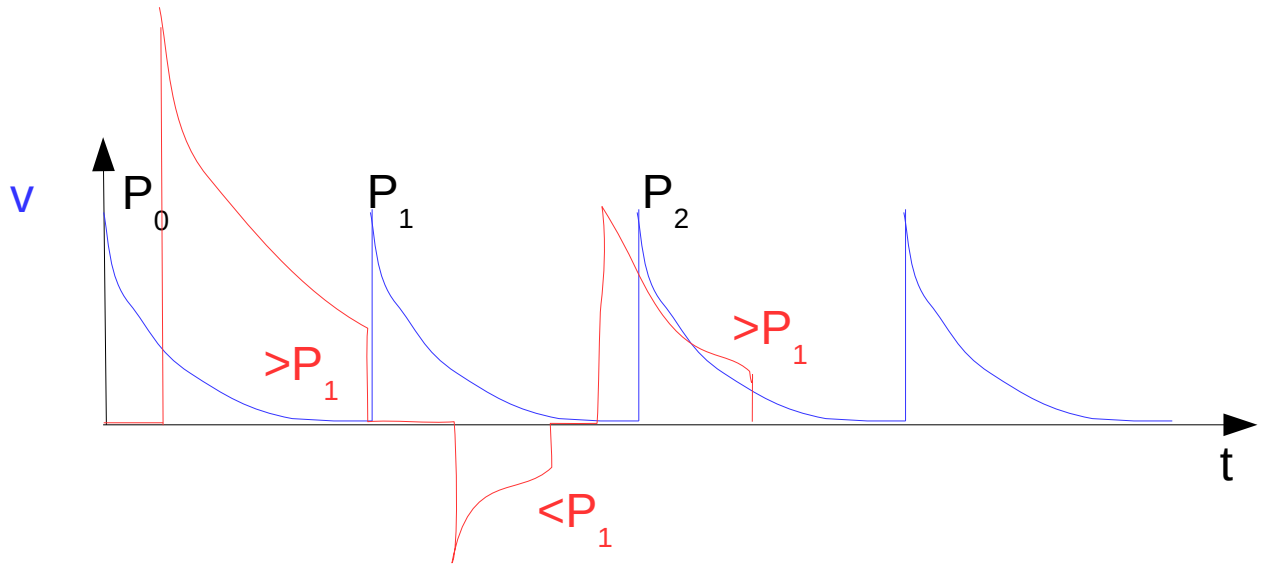
MagPier experience :

Small robot in the air over a surface = **dry friction**

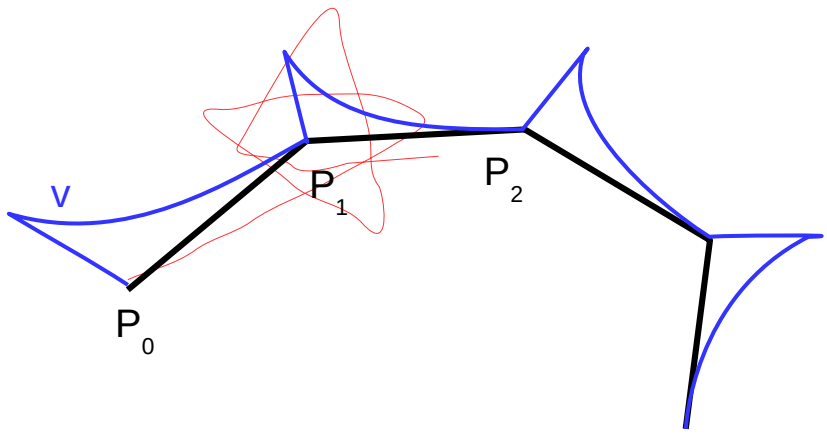


ICRA/NIST Challenge 2011 : MagPier @ ISIR/FEMTO-ST

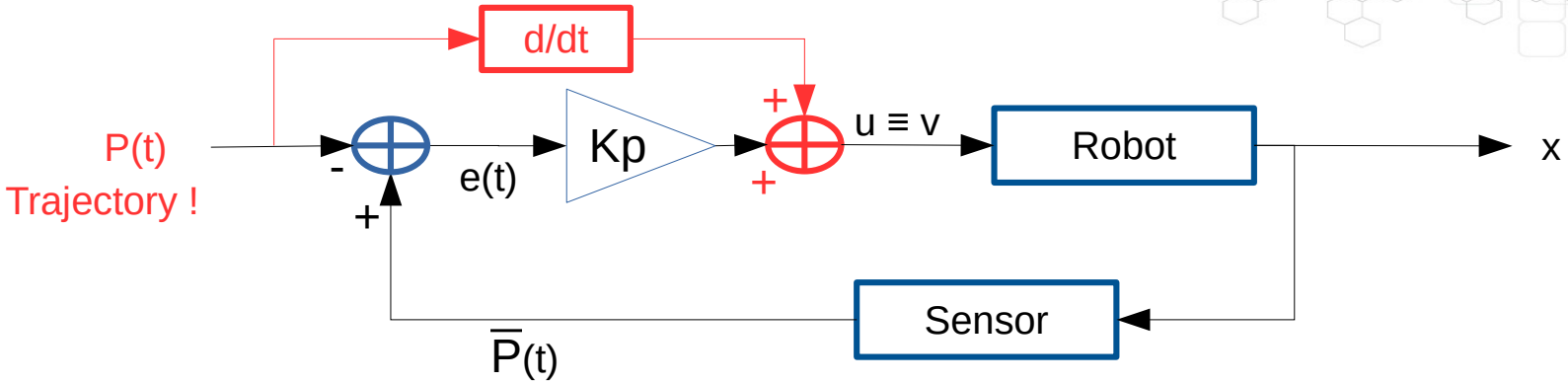
Time response



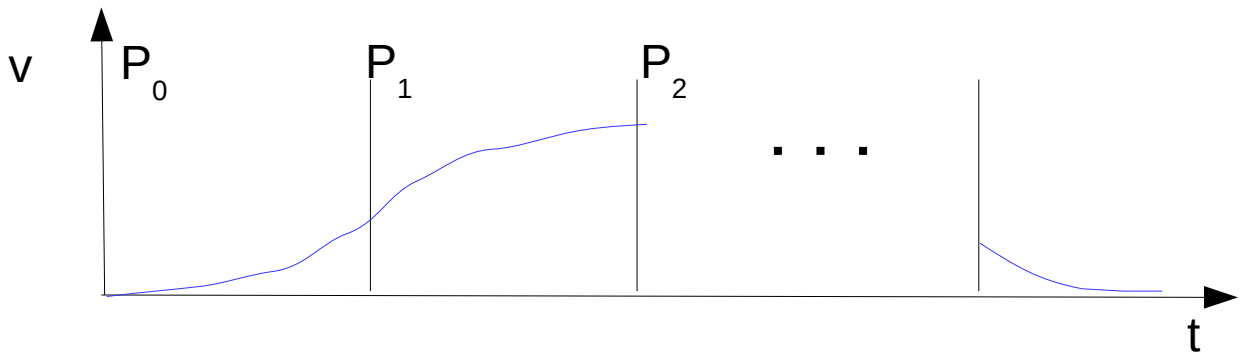
Spatio-temporal response



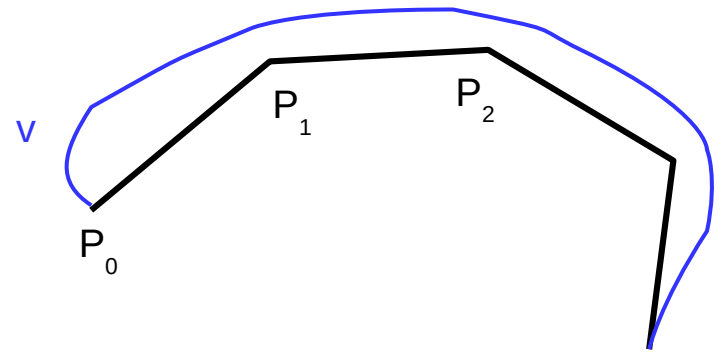
# Proportional + feedforward controller



Time response



Spatio-temporal response

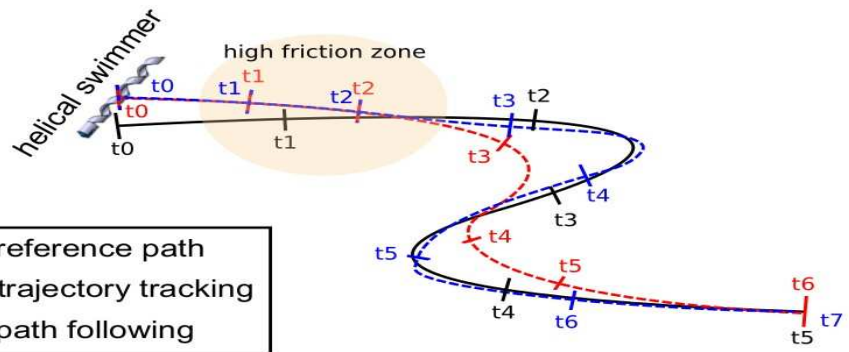
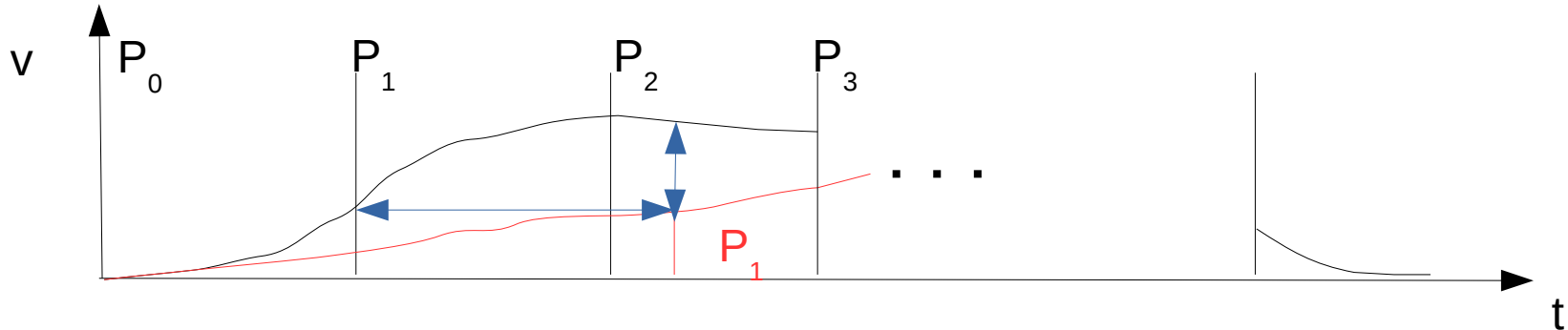


Dry friction reduced but trajectory planning is more complex than defining waypoints

# Golden Eye reloaded



Viscous/dry friction → time delay →  $P_2$  was missed → now aiming at  $P_3$

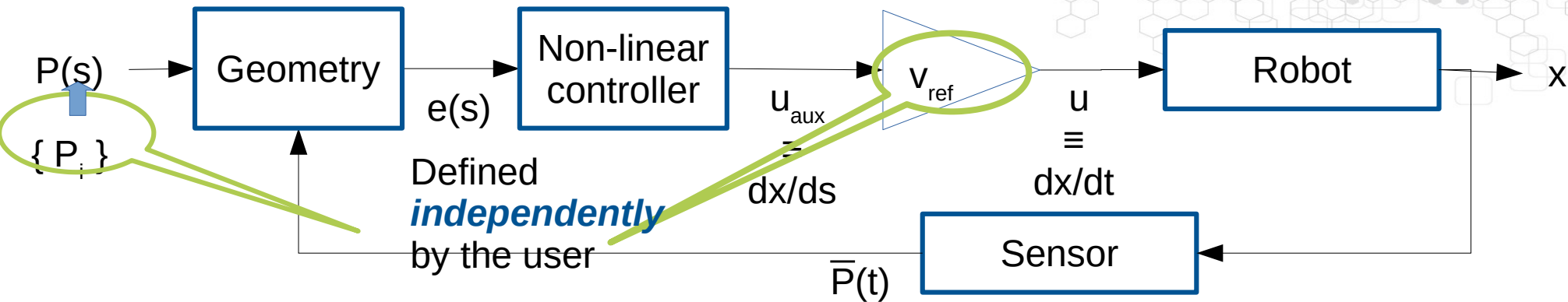


- reference path
- - - trajectory tracking
- - - path following

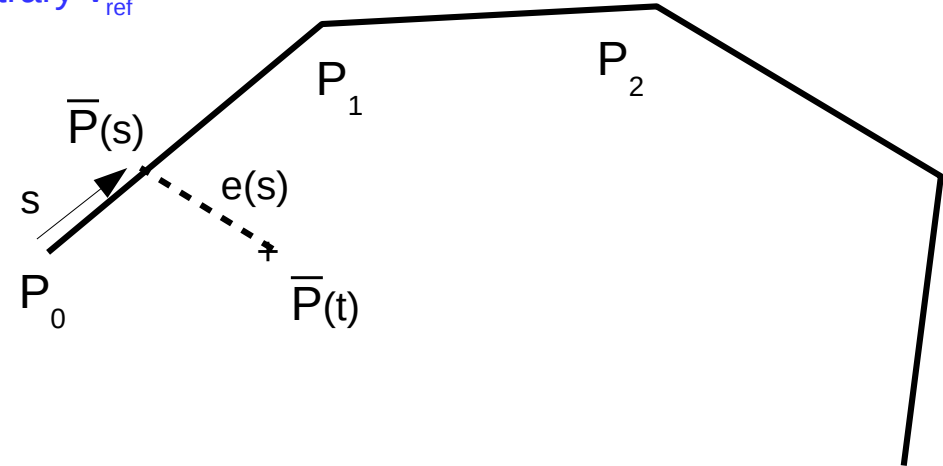
Time is the problem !



# A controller respecting the geometry



arbitrary  $v_{ref}$



- Time-independent error
- Arbitrary velocity
- ⇒ **Time and space are decoupled**
- ⇒ **User need satisfied**
- ⇒ but NL controller to be defined

# Non holonomic vehicle control



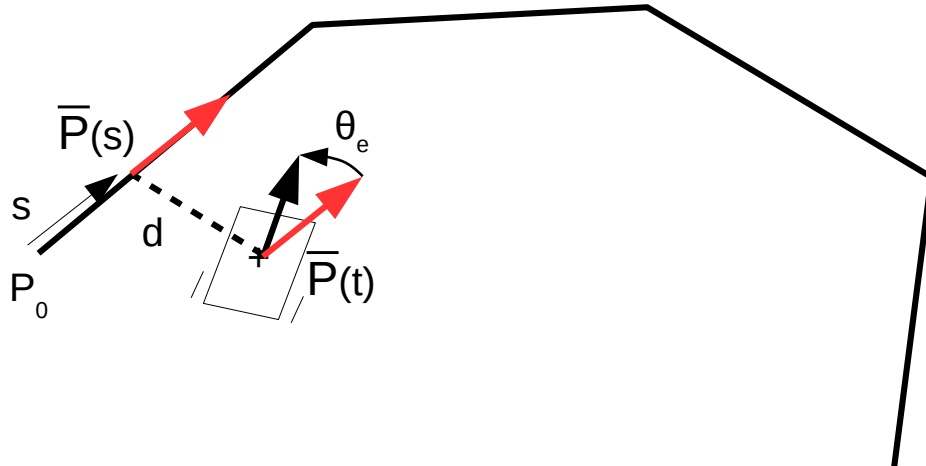
Vehicle state :  $s, d, \theta_e$

Vehicle control inputs :  $u_1 = \|v\|$   $u_2 = \omega$

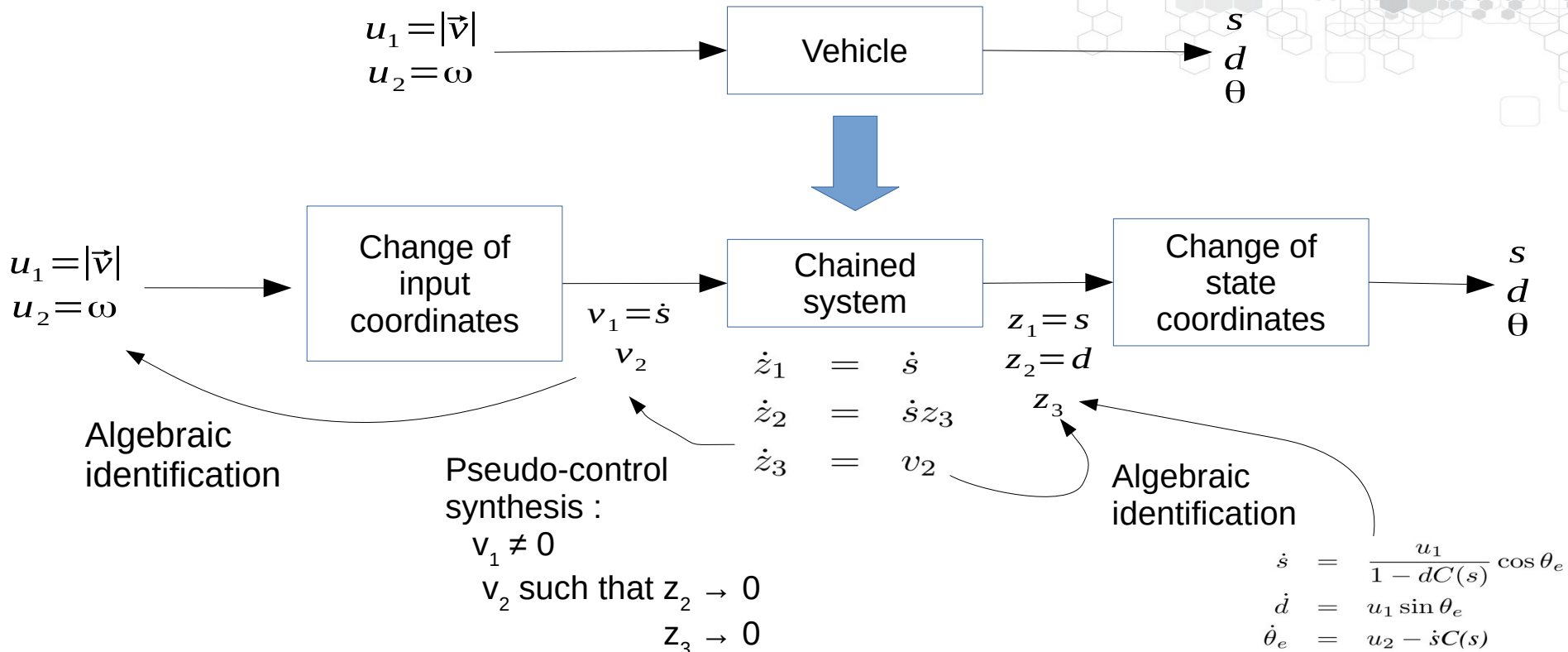
Vehicle kinematics :

$$\begin{aligned}\dot{s} &= \frac{u_1}{1 - dC(s)} \cos \theta_e \\ \dot{d} &= u_1 \sin \theta_e \\ \dot{\theta}_e &= u_2 - \dot{s}C(s)\end{aligned}$$

Non-linear  
Time-dependent  
Non-holonomic

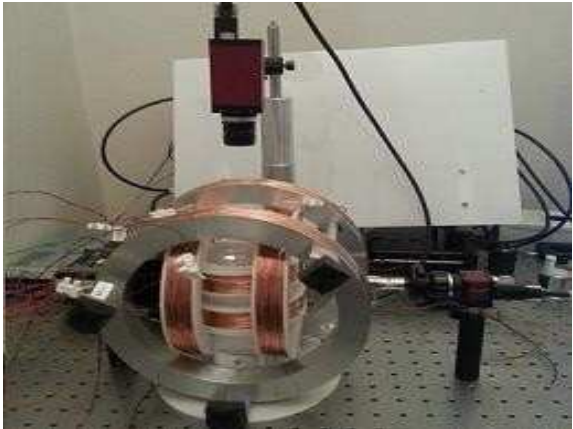


# Exact linearisation and cascade control

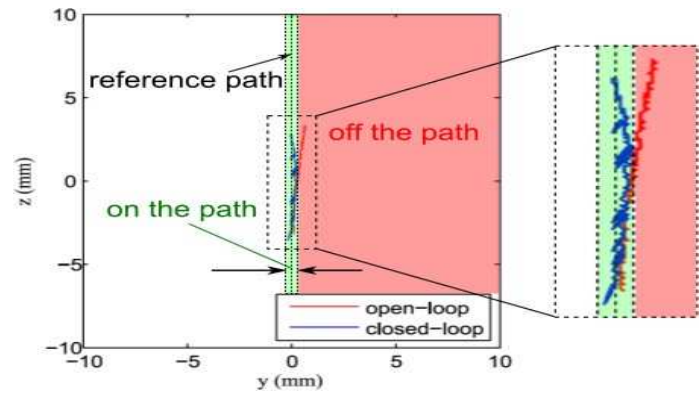
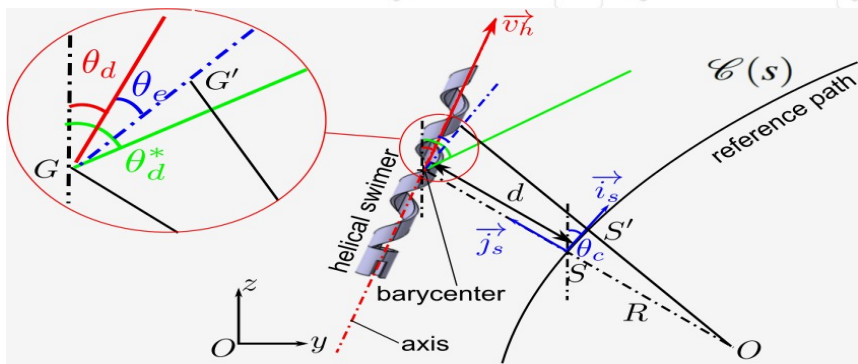


This is how you drive : wheel and throttle decoupled in normal conditions (e.g. cruise control)

# Scaled-up helical swimmer in 2D



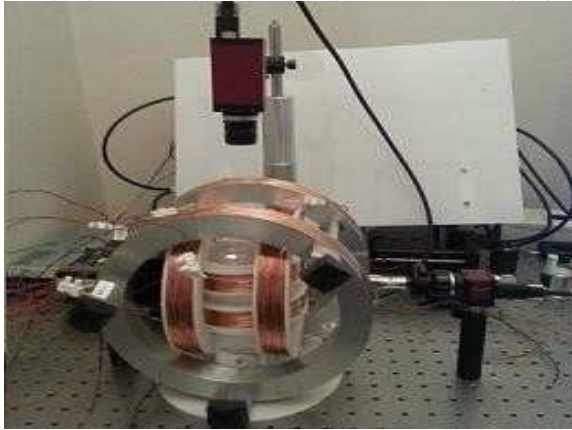
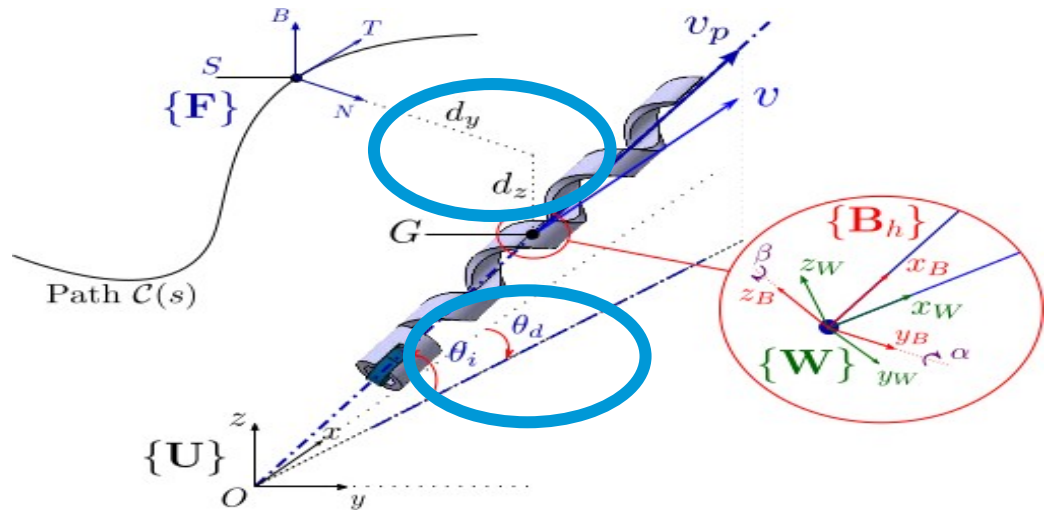
Helical swimming is non holonomic



open-loop control of the height + closed-loop control in the horizontal plane

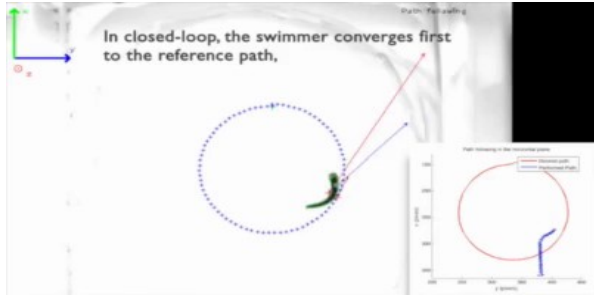
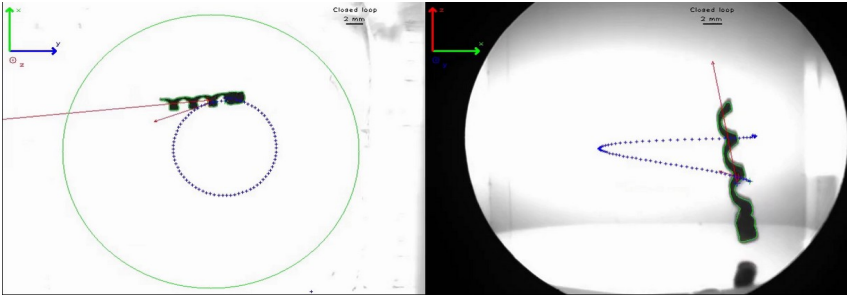
[Xu 14]

# Extension to magnetic swimming in 3D



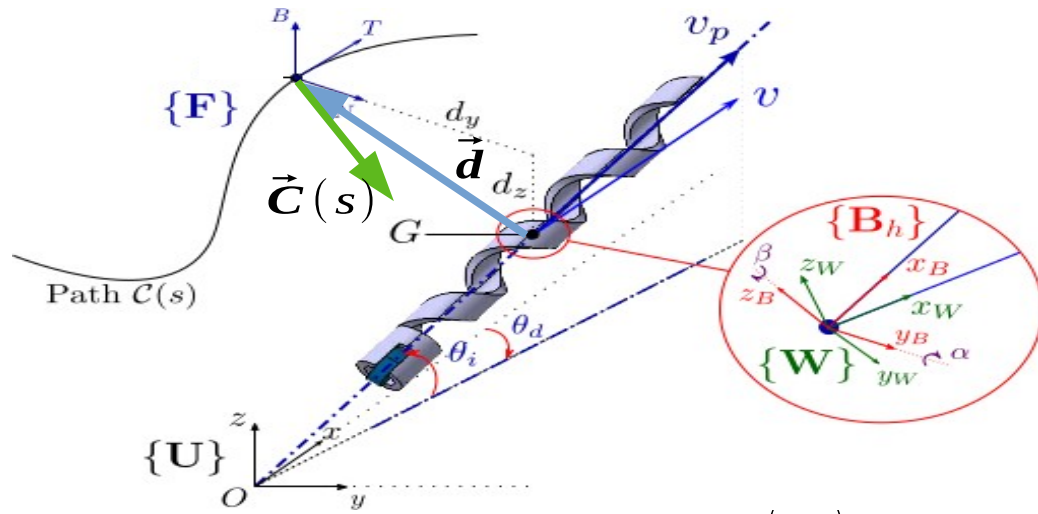
From a 3-state  $(s, d, \theta)$  2-input  $(|v|, \omega)$  chained system  
 to a 5-state  $(s, dx, dy, \theta_x, \theta_y)$  3-input  $(|v|, \omega_x, \omega_y)$  chained system

+ independence from propulsion mode



[Oulmas 18]

# Towards a simpler formalism



State : Algebraic analysis (incl. curvature)  $\begin{pmatrix} s \\ \vec{d} \\ \vec{v}_p \end{pmatrix}$

Input : Geometric controller  $\vec{u}_1 = u_1 \vec{v}_p$

→ Lyapunov stability ?

$$\dot{\vec{d}} = \left( \mathbf{I} - \frac{\vec{x}_s \vec{x}_s^T}{1 - \vec{d}^T \vec{C}(s) \times \vec{T}} \right) \vec{u}_1$$

$$\vec{u}_1 = \alpha \vec{T} + \beta \vec{d}$$

$$|\vec{u}_1| = \text{arbitrary} \Rightarrow \alpha = \sqrt{|\vec{u}_1|^2 - \beta^2 |\vec{d}|^2}$$

# Take home messages

- **A microswimmer is not a robot**
- **A new class of magnetic manipulation systems**  
multi-mobile electromagnet systems
- **Several modes of actuation**  
Field only, Force/Field, (Force only)
- **Two control strategy**  
trajectory tracking, path following
- **A control-oriented magnetic model**  
fast and accurate enough
- **Typical robotic issues**  
Manipulability, dexterity, singularities





