

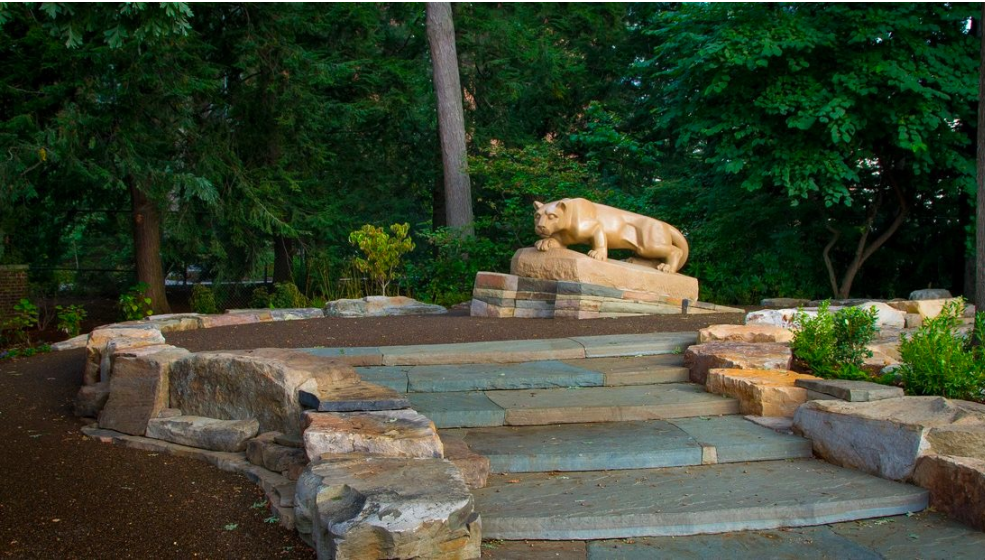
# **Spatiotemporal organization of a living nematic by confinement and surface anchoring**

Igor Aronson  
Pennsylvania State University

PENNS<sup>T</sup>ATE<sup>®</sup>



# Penn State University



- State College: equally close to New York, Washington DC, Philadelphia, and Pittsburg (3 hours by car)
- twenty-four campuses (main campus 45,000 students)
- 17,000 faculty and staff
- >100,000 students
- second largest football stadium in the US (112,000 seats)
- operating budget >7 billion dollars

# Active Matter

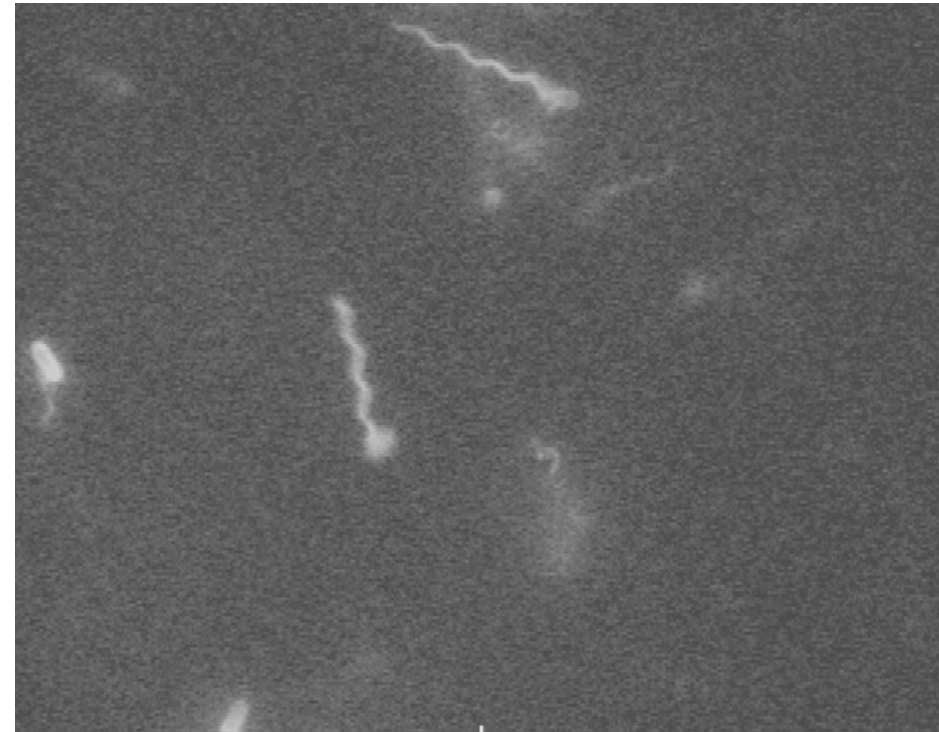
- Active Matter - a wide class of systems actively consuming energy from environment, such as assemblages of active self-propelled particles. The particles have a propensity to convert energy stored in the medium to motion
- Examples: suspensions of swimming bacteria and synthetic microswimmers, cytoskeletal actin-myosin networks, driven colloids
- Active materials exhibit properties not available at equilibrium: self-healing, adaptation, shape change

# Suspension of microswimmers: one of the simplest realizations of active matter

Bacillus subtilis



- about 5  $\mu\text{m}$  long, 0.7  $\mu\text{m}$  diameter
- swim up to 20  $\mu\text{m}/\text{s}$
- low tumbling rate
- aerobic (need oxygen to swim)



R Goldstein, Cambridge, UK  
M Graham, U Wisc  
E Clement, ESPCI, France  
H-P Zhang, Shanghai JT Univ  
M Shelley, Courant  
D Saintillan, UIUC/UCSD  
Holger Stark, TU Berlin  
Hartmut Löwen, U Dusseldorf  
T Ishikawa, Tohoku U  
A Beer, Ber Sheva, Israel

Ryu, Turner, Berg, Harvard Univ

# A living nematic

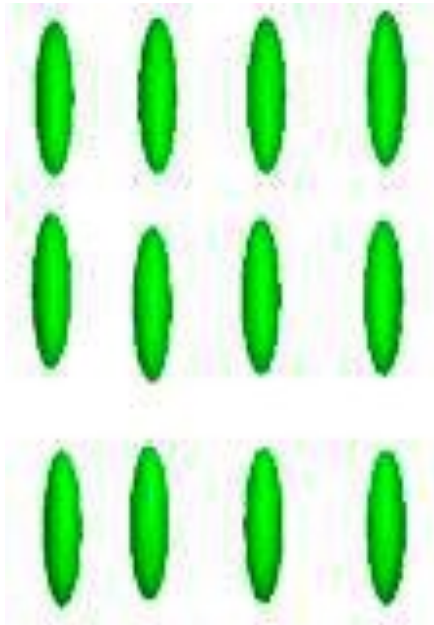


# Liquid crystals

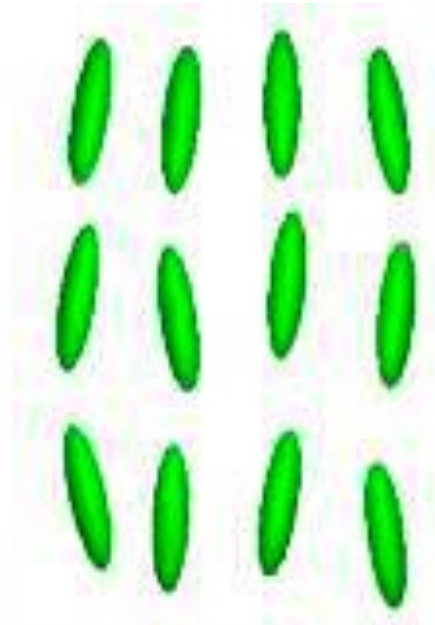
*Orientational  
and positional  
order*

*Orientational order*

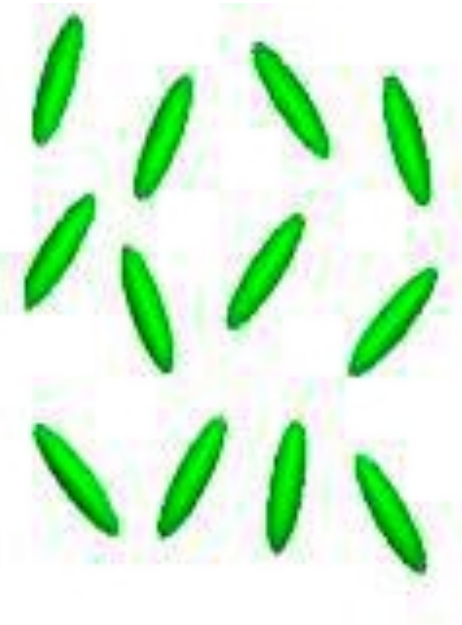
*No order*



**Solid**



**Liquid Crystal**



**Liquid**

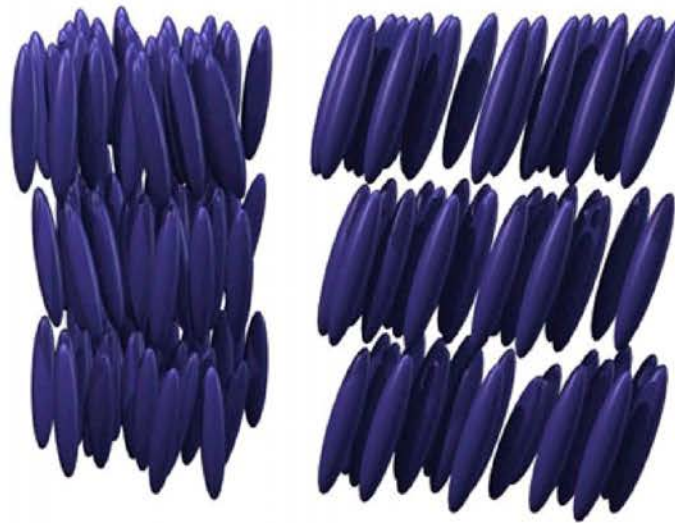
# liquid crystal phases

*orientational  
order: nematic*



(a)

*layering:  
smectic*



(b)

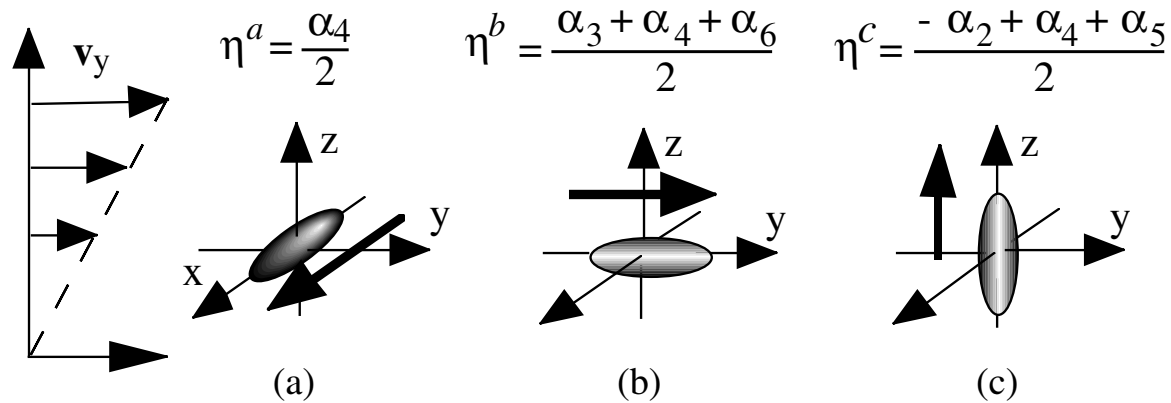
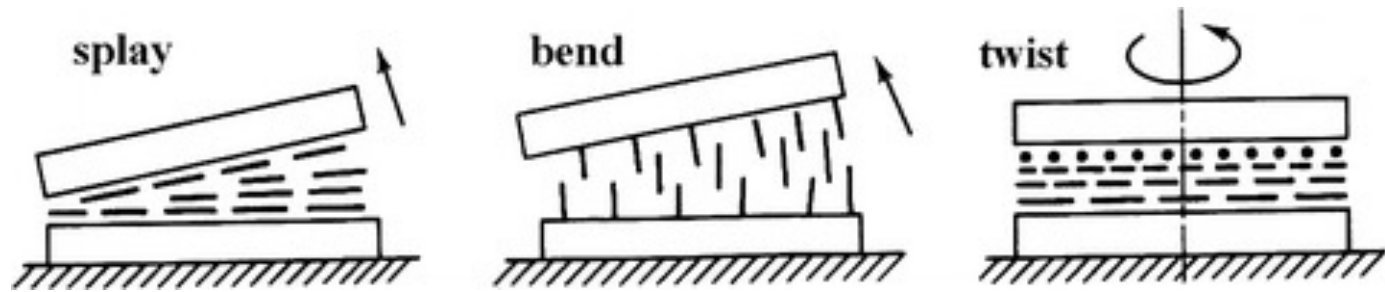
*chiral order:  
cholesteric*



(c)

**$\mathbf{n}$**  *nematic director:*  
*(apolar) vector along molecular axis*

# liquid crystal deformations



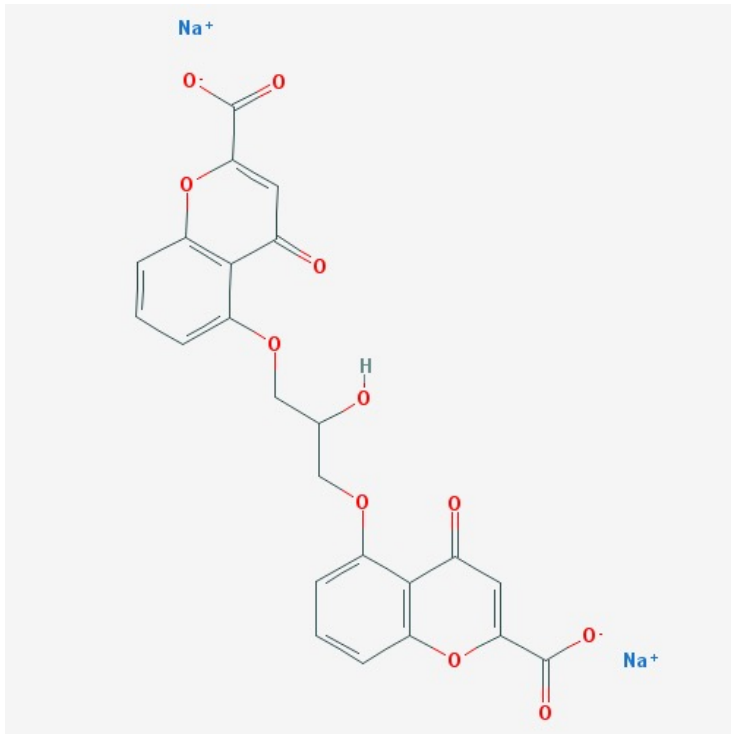
*nematic liquid crystals (DSCG)*

3 elastic constants  $K_{1,2,3}$  (splay, bend  $\gg$  twist)

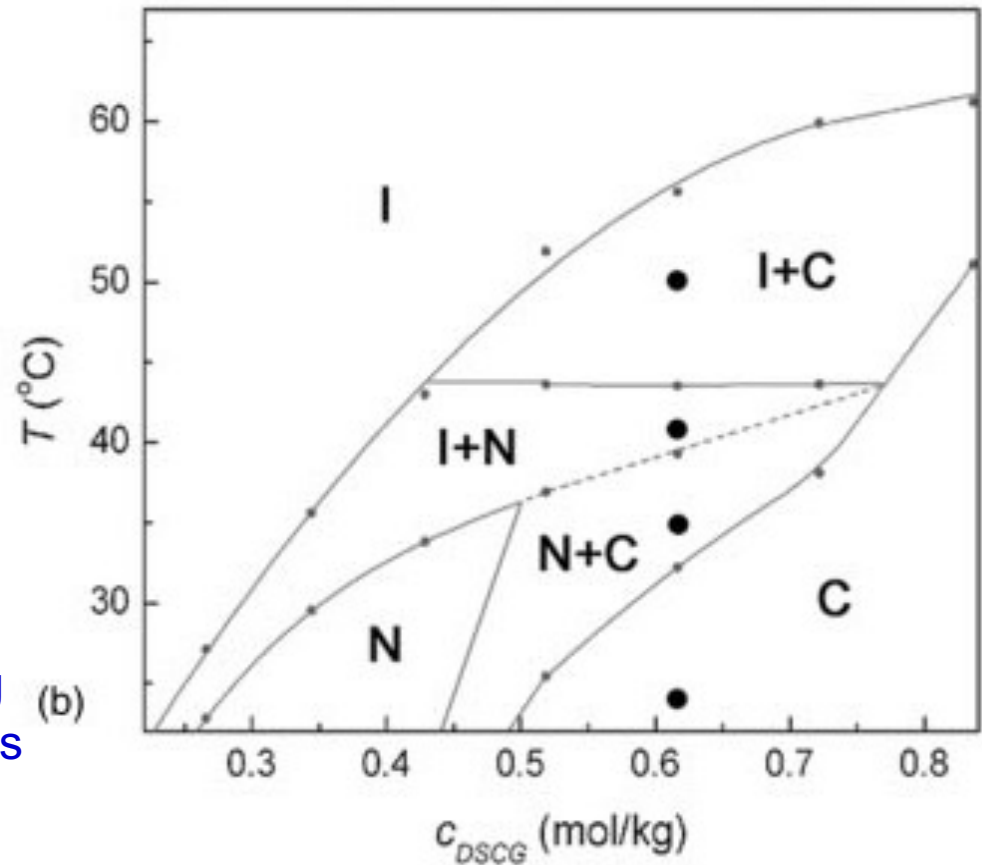
3 viscosities:  $\eta_{a,c} \sim 10^4 \eta_b$



# LC: DSCG



## Phase Diagram



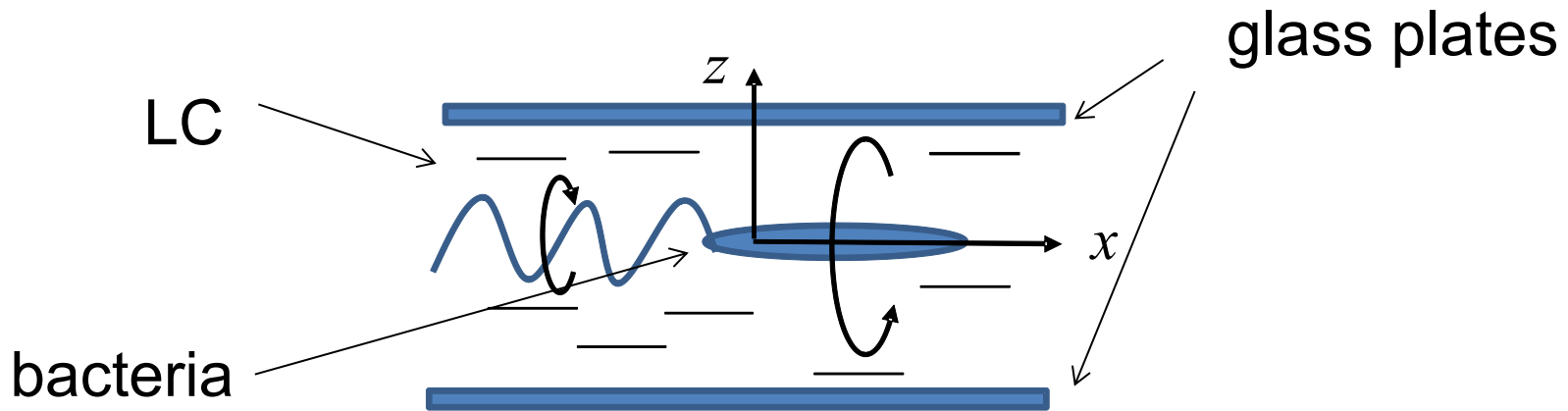
DSCG, a.k.a. cromolyn, anti-asthma drug  
disc-like molecules from linear aggregates

- I – isotropic
- N – nematic
- C – columnar

# Mucus – biological liquid crystal

- protective, exchange, and transport medium in the digestive, respiratory and reproduction systems
- Protects against infectious agents such as fungi, bacteria, and viruses
- Better understanding of bacteria-mucus interaction is crucial for the study and development of better treatment of many diseases
- Long molecules form liquid crystalline phase of mucus

# Mark I Experimental Cell

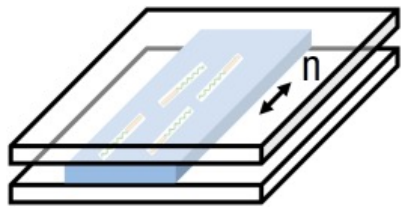


- nematic LC
- planar director surface anchoring (x-direction)
- plates coated with polyimide
- rubbed with a velvet cloth
- thickness : 5 – 500 microns
- concentration of bacteria: 0.2 – 3%
- temperature 25-35 C

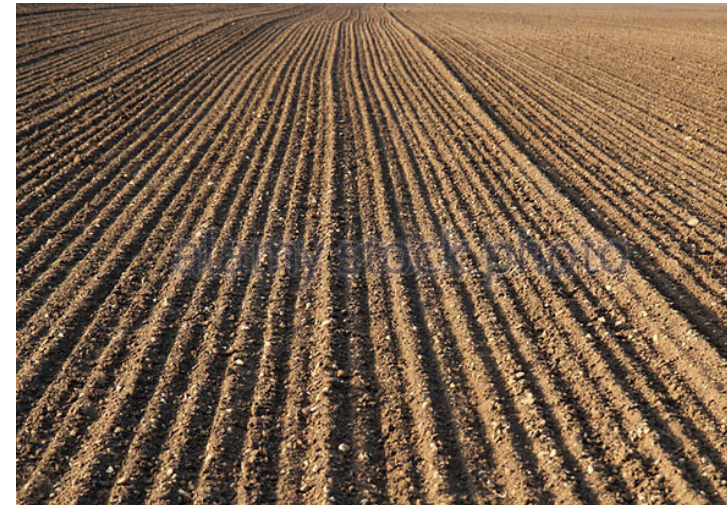
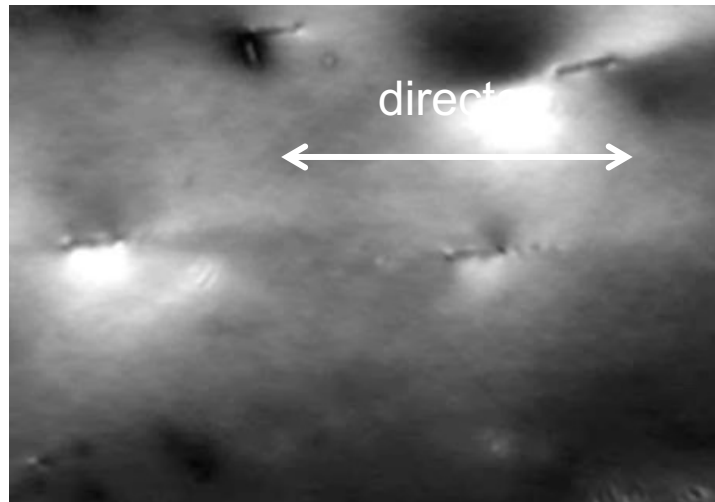
# Bacteria follow director of a LC (low concentration)

director (non-polar vector) – average molecular orientation

flat glass cell



Thickness  
 $h = 5-10 \mu\text{m}$

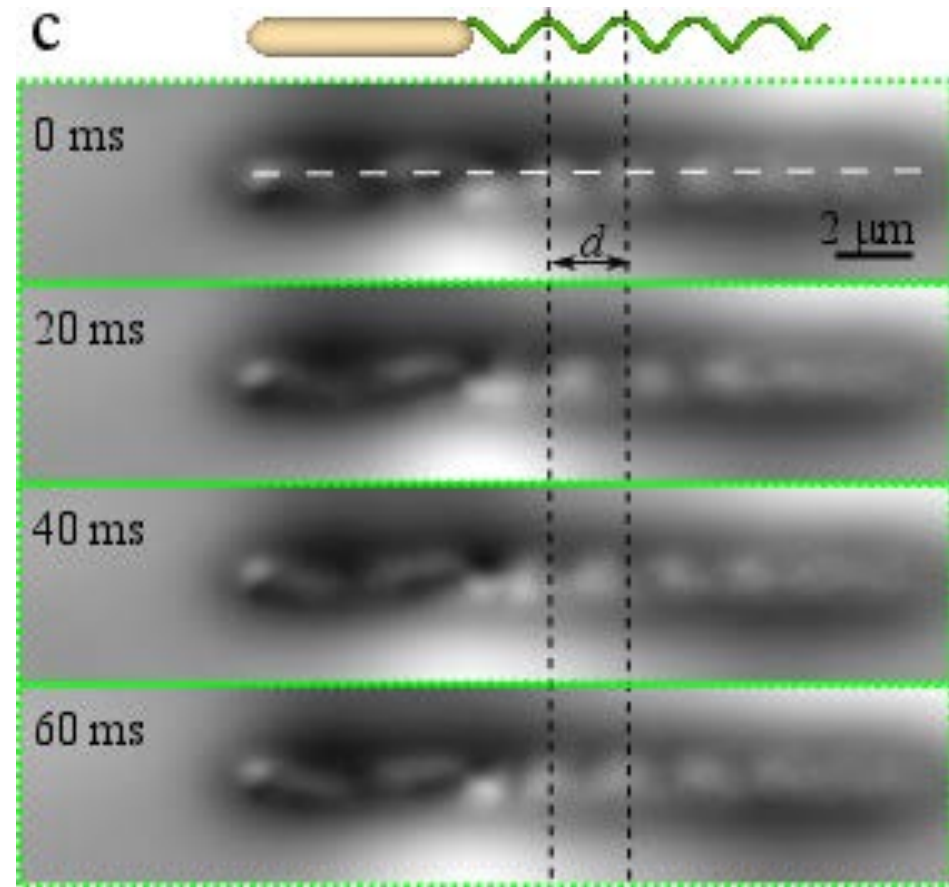


# Zoom on individual bacterium

direct optical visualization  
of the 24 nm flagella!

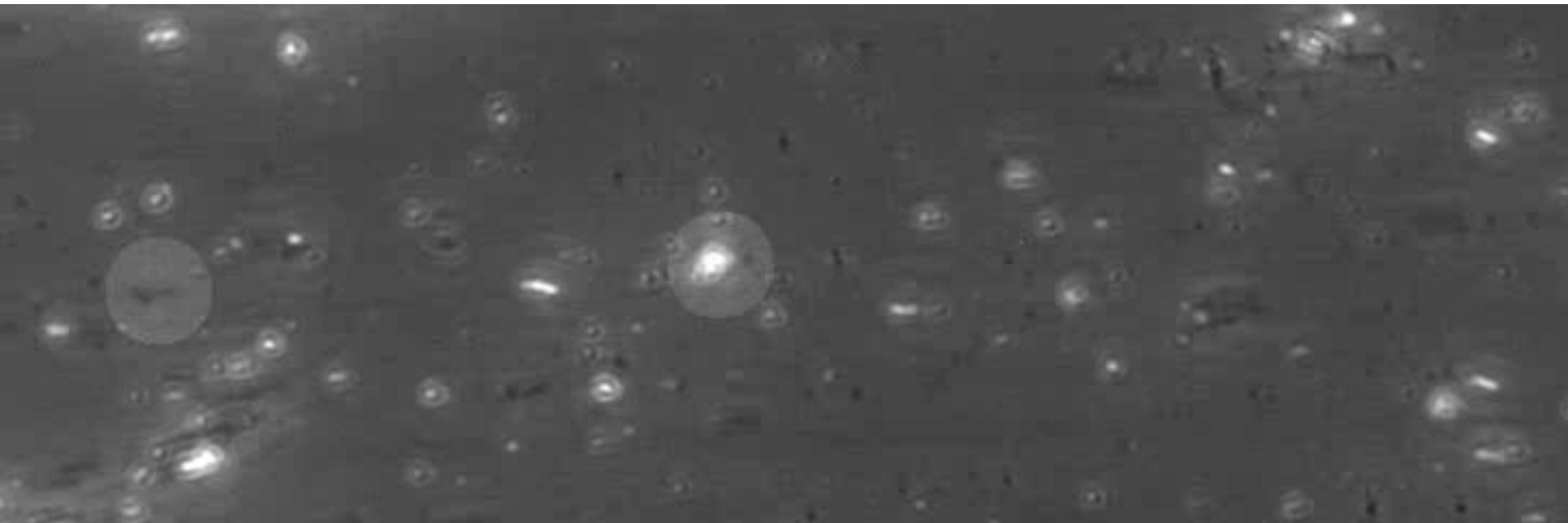


- flagella rotation – 10 Hz
- body counter-rotation – 2 HZ



# Tracer-bacterium interaction: targeted cargo transport and delivery

Evidence for the long-range interaction

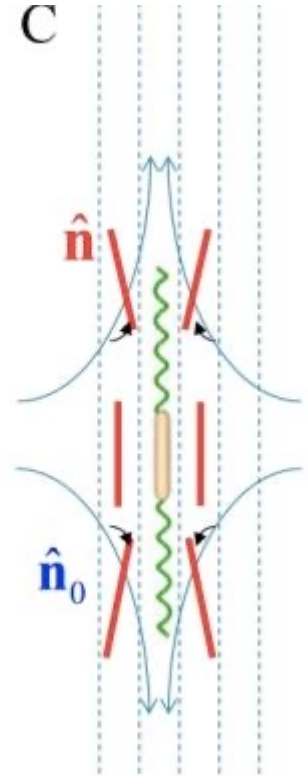
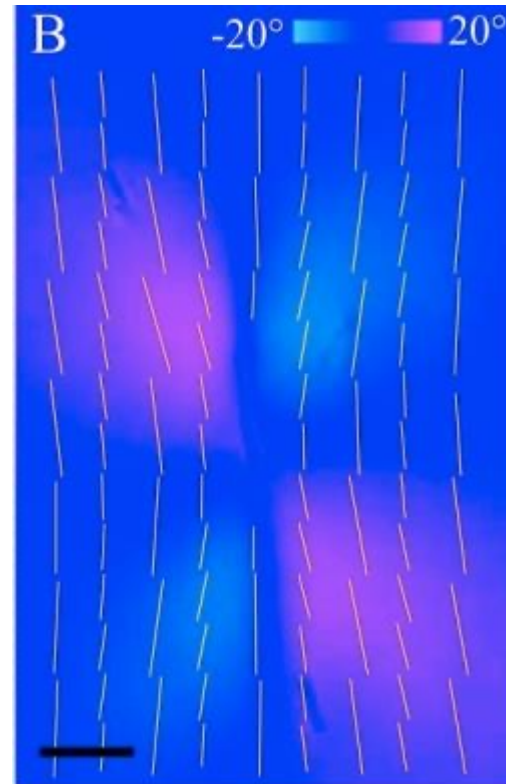


# Direct flow visualization flow → butterfly pattern

Crossed-polarized image

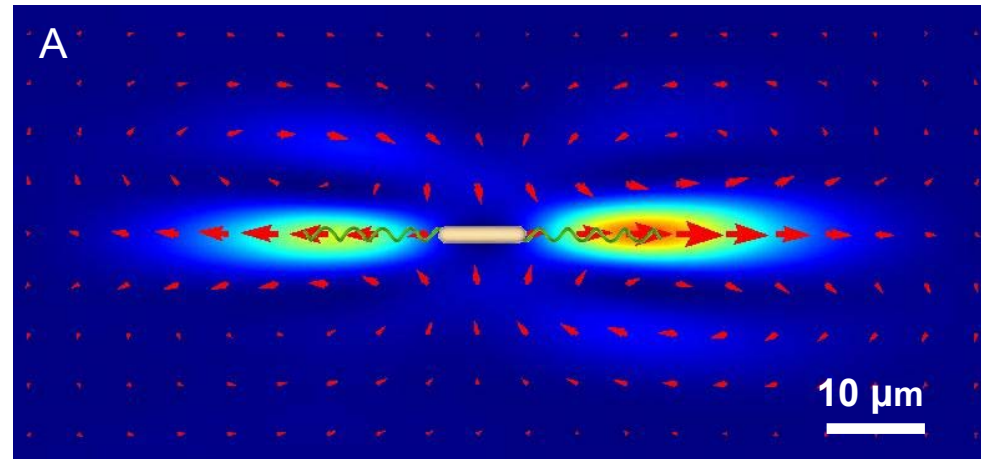


Polscope image

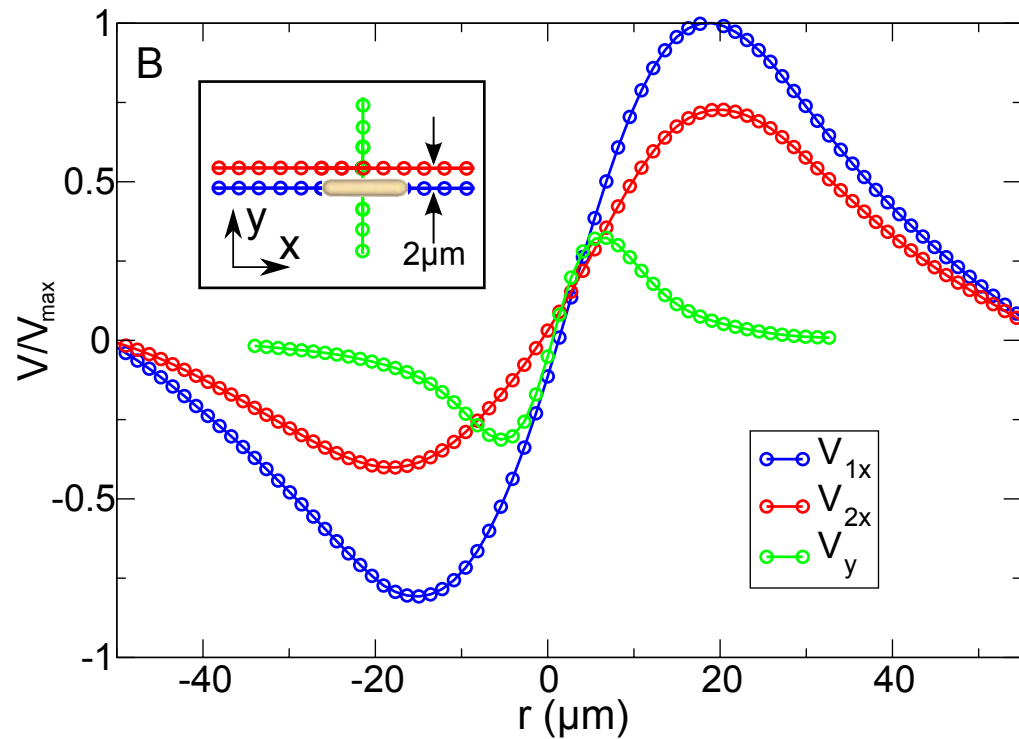


polscope directly visualizes optical retardance

# Reconstructed flow field



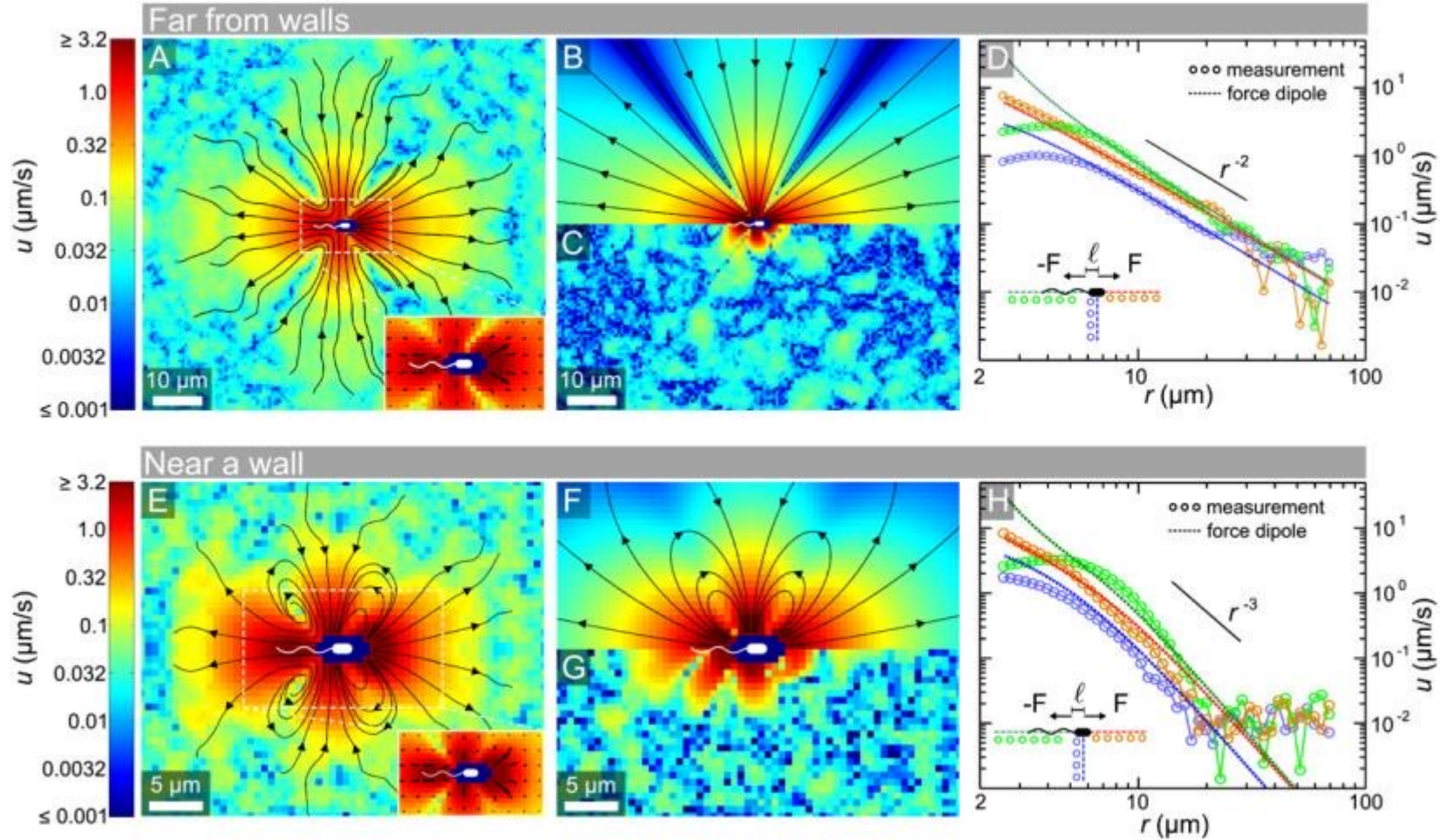
Flow field is reconstructed from the Leslie-Ericksen equation





# Average flow field created by a single freely swimming bacterium far from surfaces (A–D) and close to a wall (E–H).

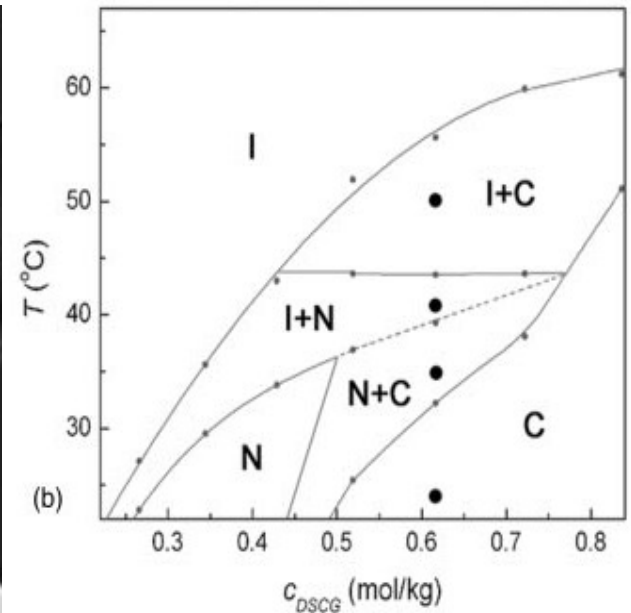
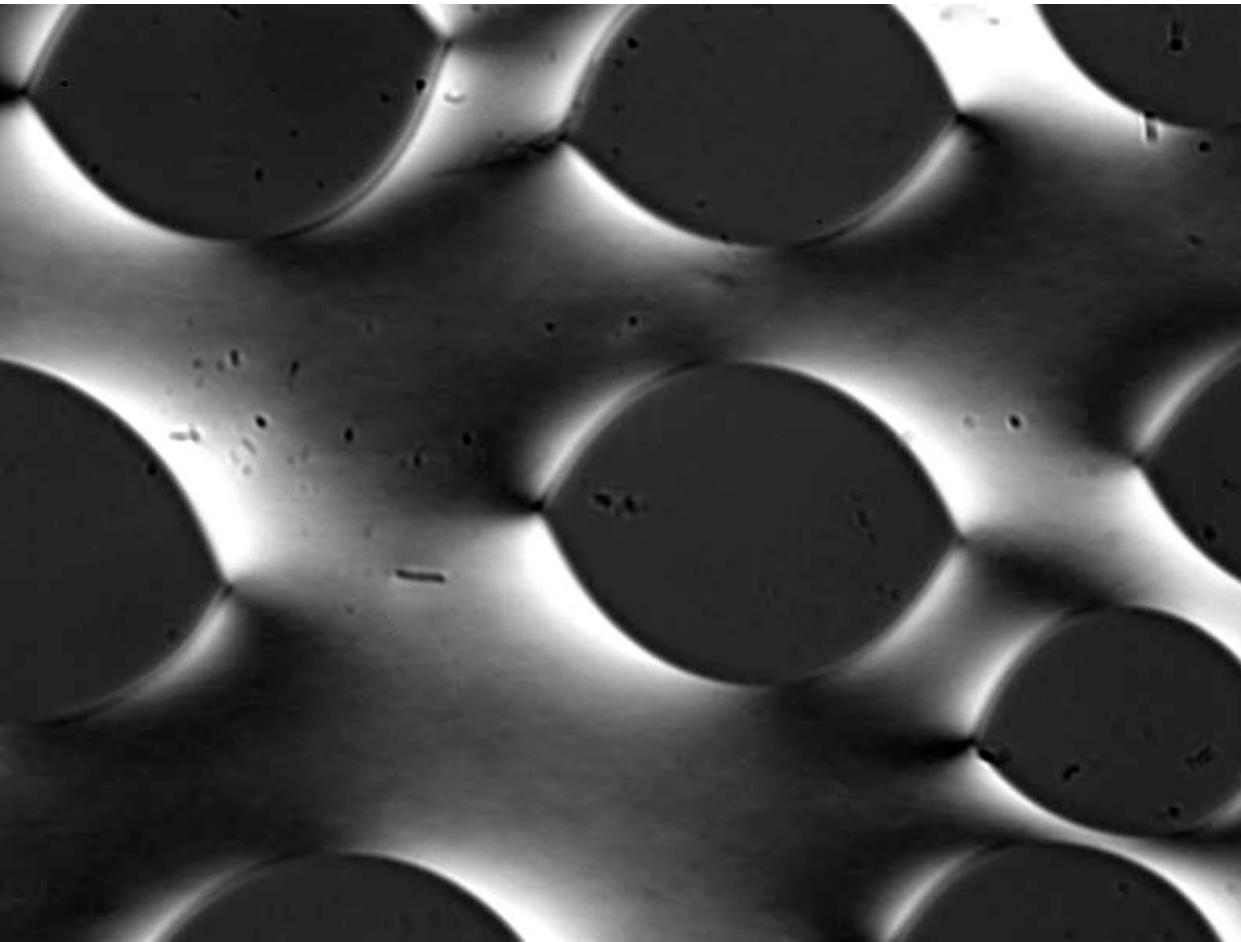
Induced flow comparable to the swimming speed at a distance approx 0.5 microns



Drescher K et al. PNAS 2011;108:10940-10945

# Guidance of bacteria in a biphasic domain

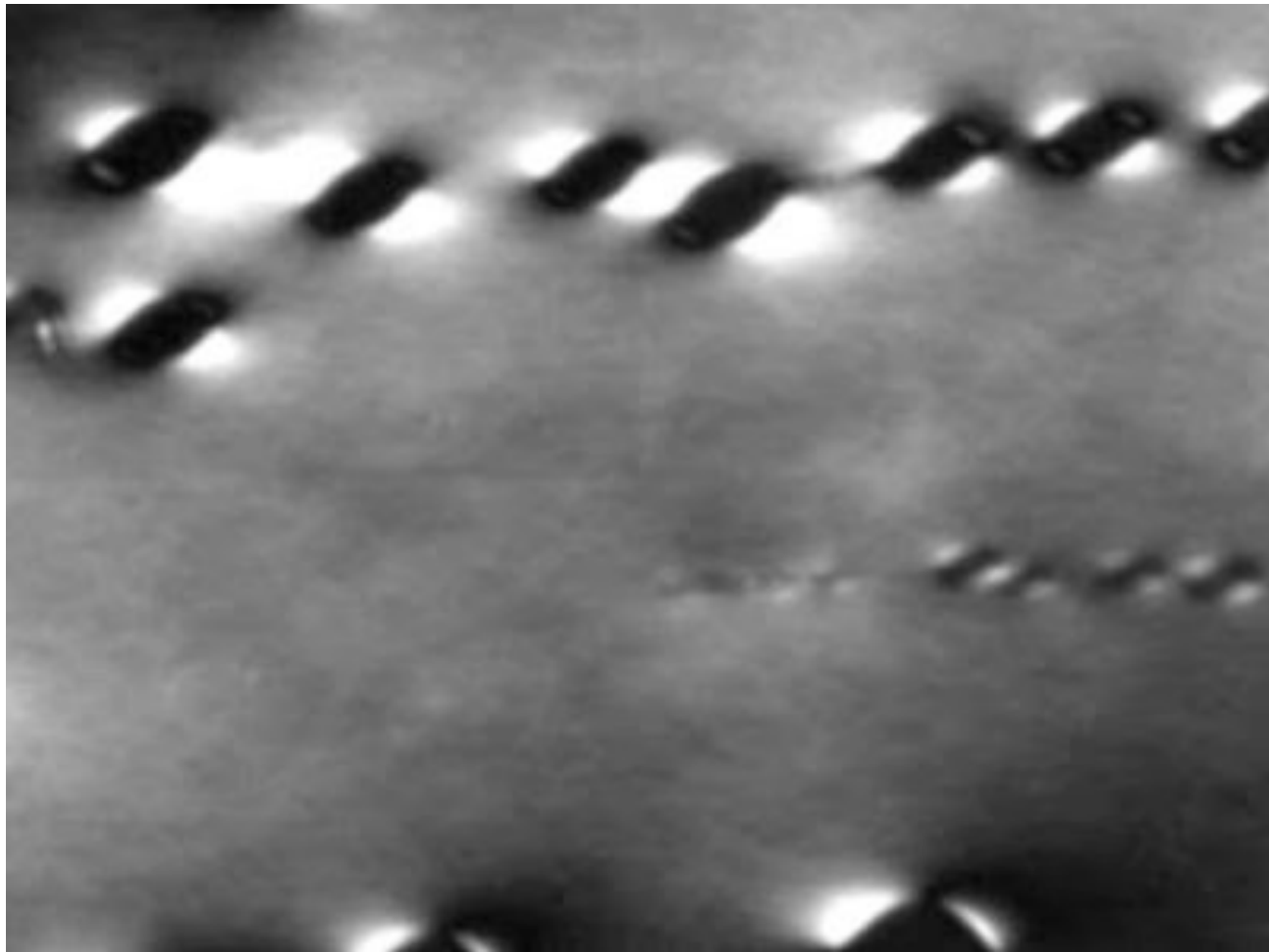
higher temperature – nematic/isotropic phases co-exist  
bacteria follow the boundaries of normal tactoids



# Living LC in the biphasic domain

higher temperature – nematic/isotropic phases co-exist

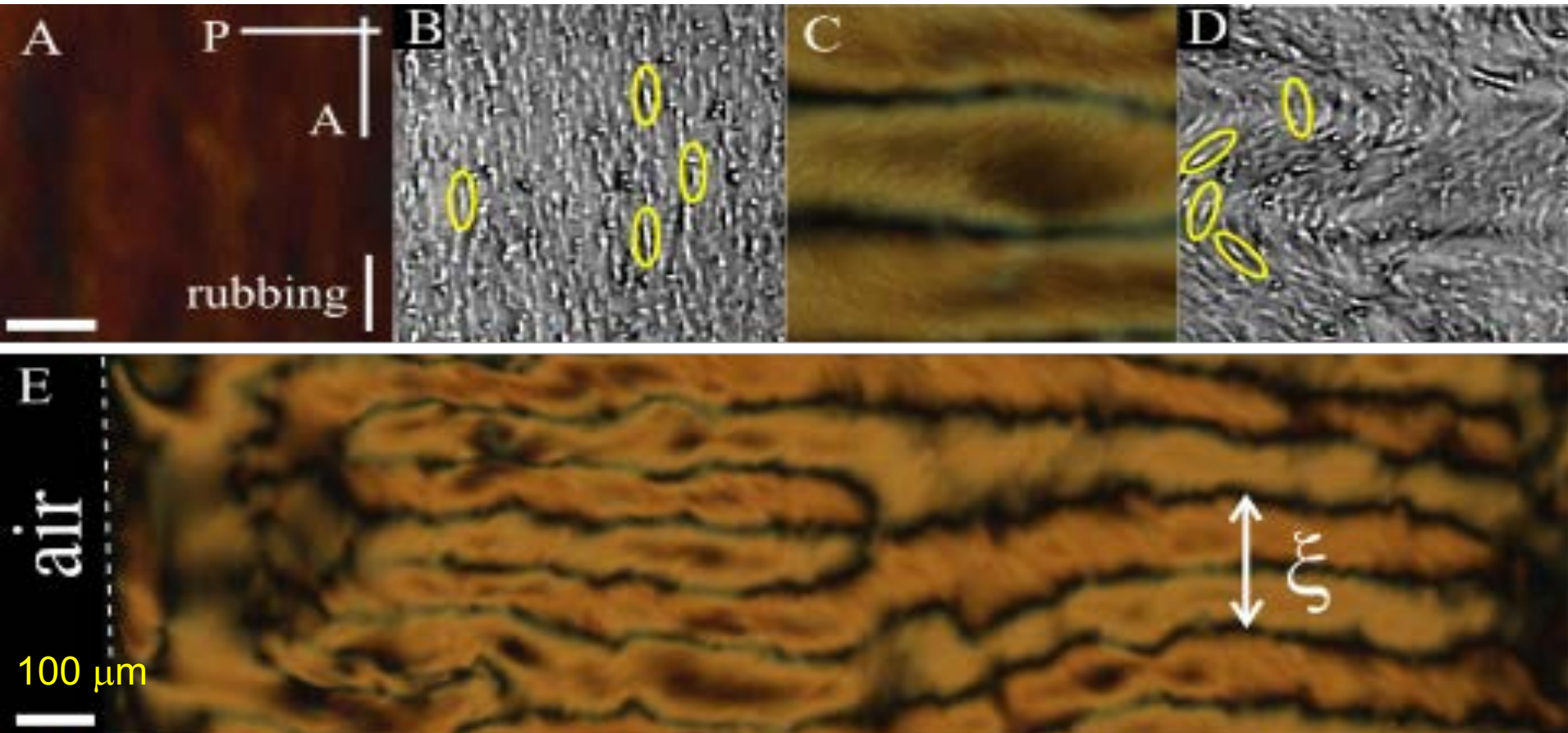
bacteria melt LC and nucleate tactoids – cloud chamber



# Higher Concentrations: Collective Effects

no oxygen: equilibrium state  
of uniform director

with oxygen: director  
undulations and stripes



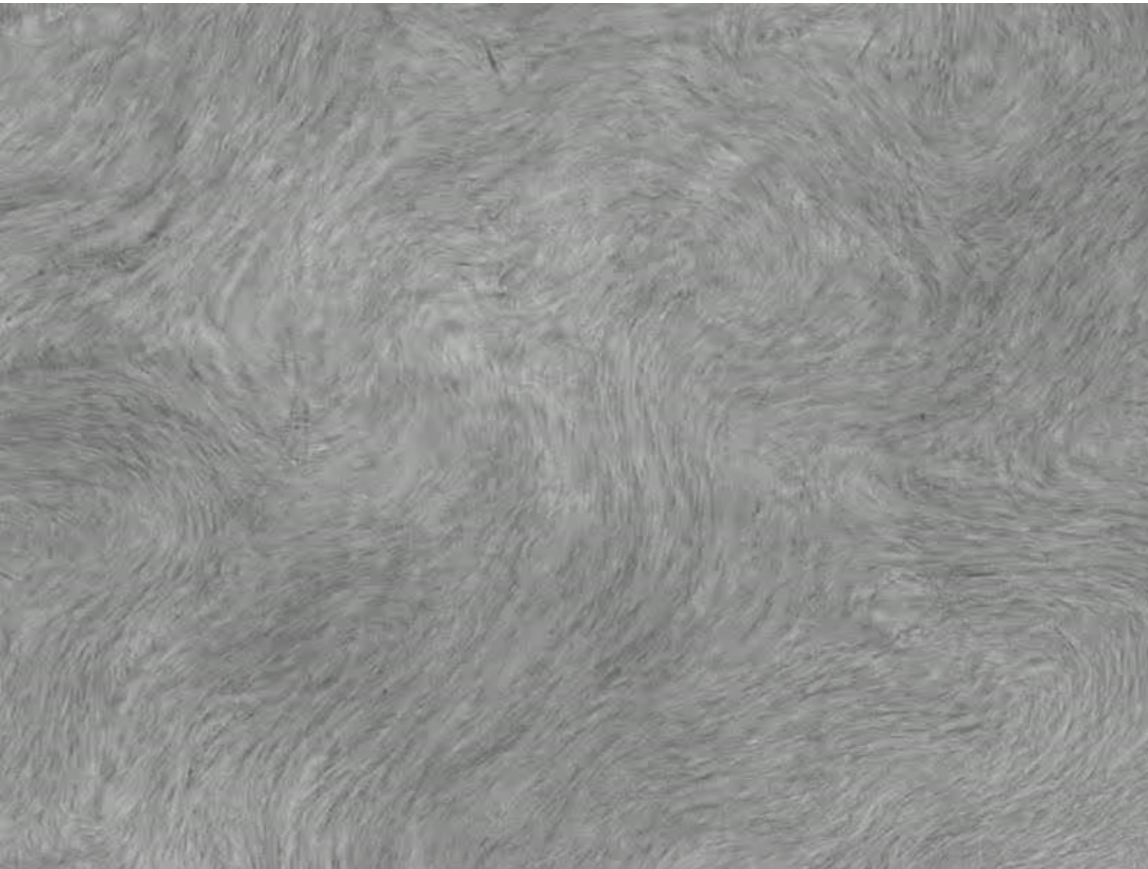
# Collective Effects: Formation of Stripe Pattern

scale depends on concentration, amount of oxygen  
extreme sensitivity-> possible biosensor applications



# Bacterial Turbulence in LC

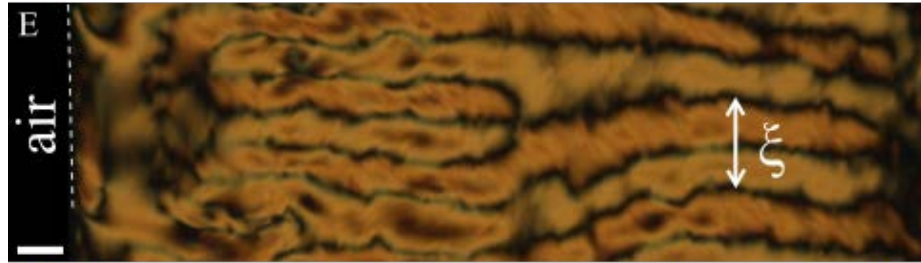
## Bacteria in LC



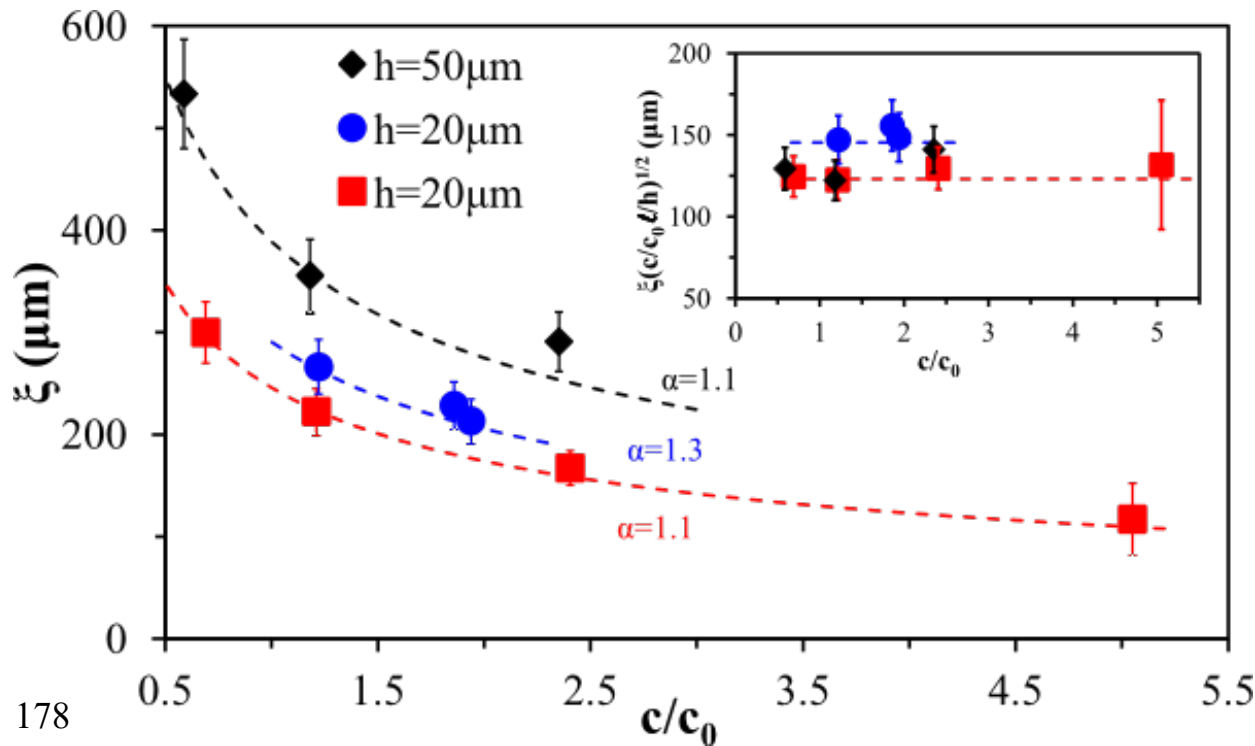
MT+ MM, Dogic  
group, Nature 2013

Active 2D nematic  
Low curvature interface  
60X mag  
15 $\mu$ m bar

# Emergence of the coherence length (period of the stripe pattern)



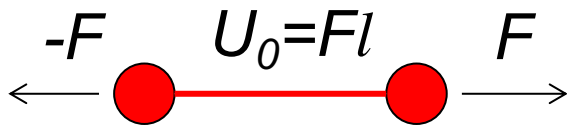
coherence length vs concentration



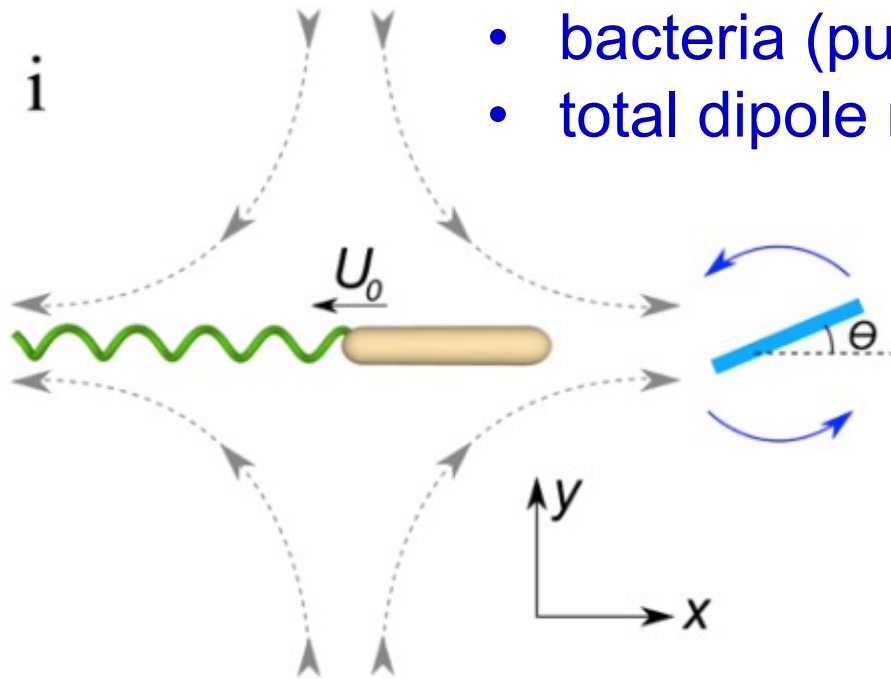
theoretical prediction

$$\xi = \sqrt{\frac{Kh}{\alpha c U_0}}$$

# Estimation of the coherence length



- bacteria (pushers) create force dipole  $U_0$
- total dipole moment of the suspension  $cU_0$



Balance of viscous and elastic (LC) torques

$$\left. \begin{array}{l} \Gamma_{shear} \sim \alpha c U_0 \theta \\ \Gamma_{elastic} \sim K \frac{\partial^2 \theta}{\partial x^2} \end{array} \right\} \Rightarrow \xi = \sqrt{\frac{K}{\alpha c U_0}}$$

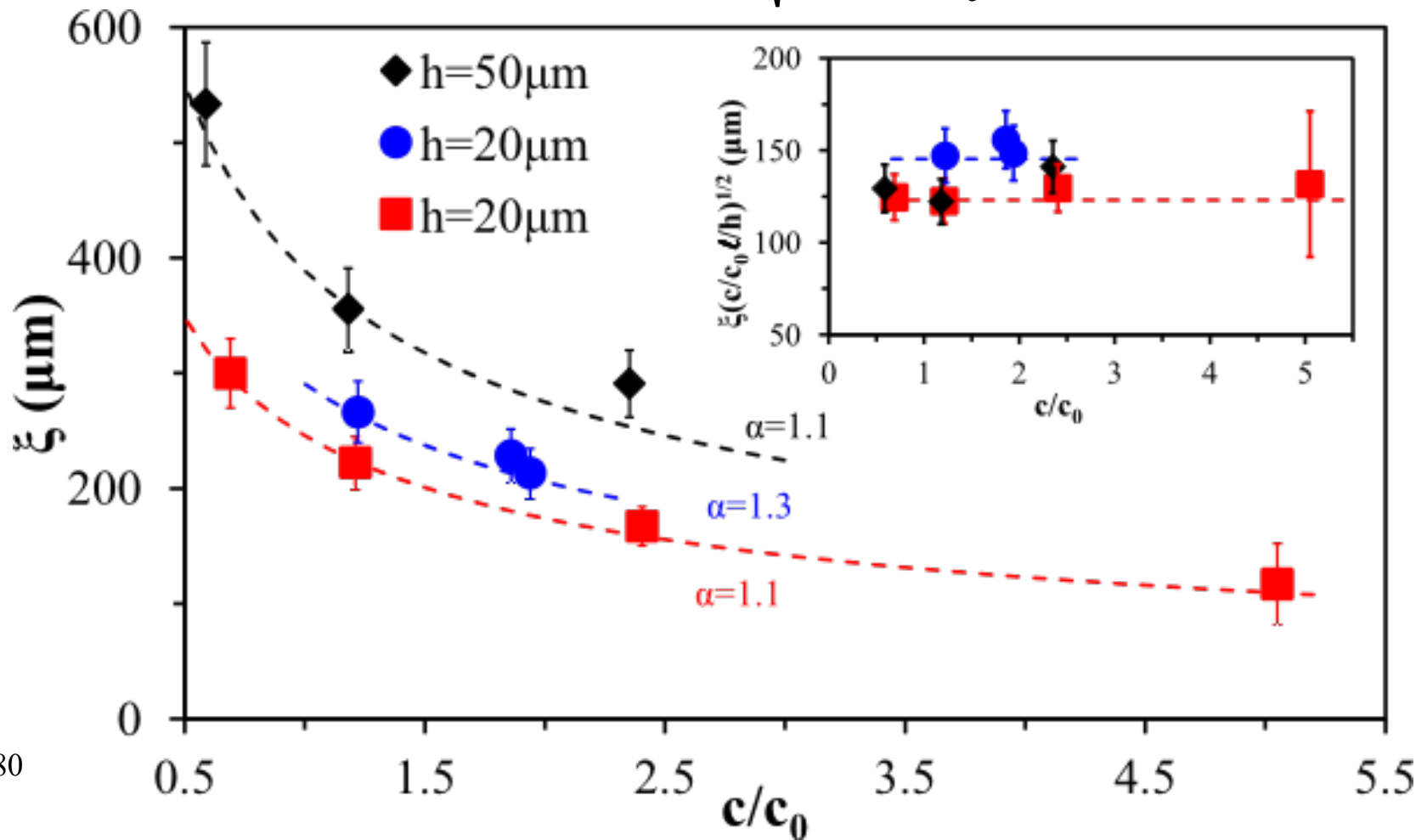
Correction due to the finite cell thickness (mass conservation)

$$\xi = \sqrt{\frac{Kh}{\alpha_0 l c U_0}}; \quad \alpha \rightarrow \alpha_0 l / h$$



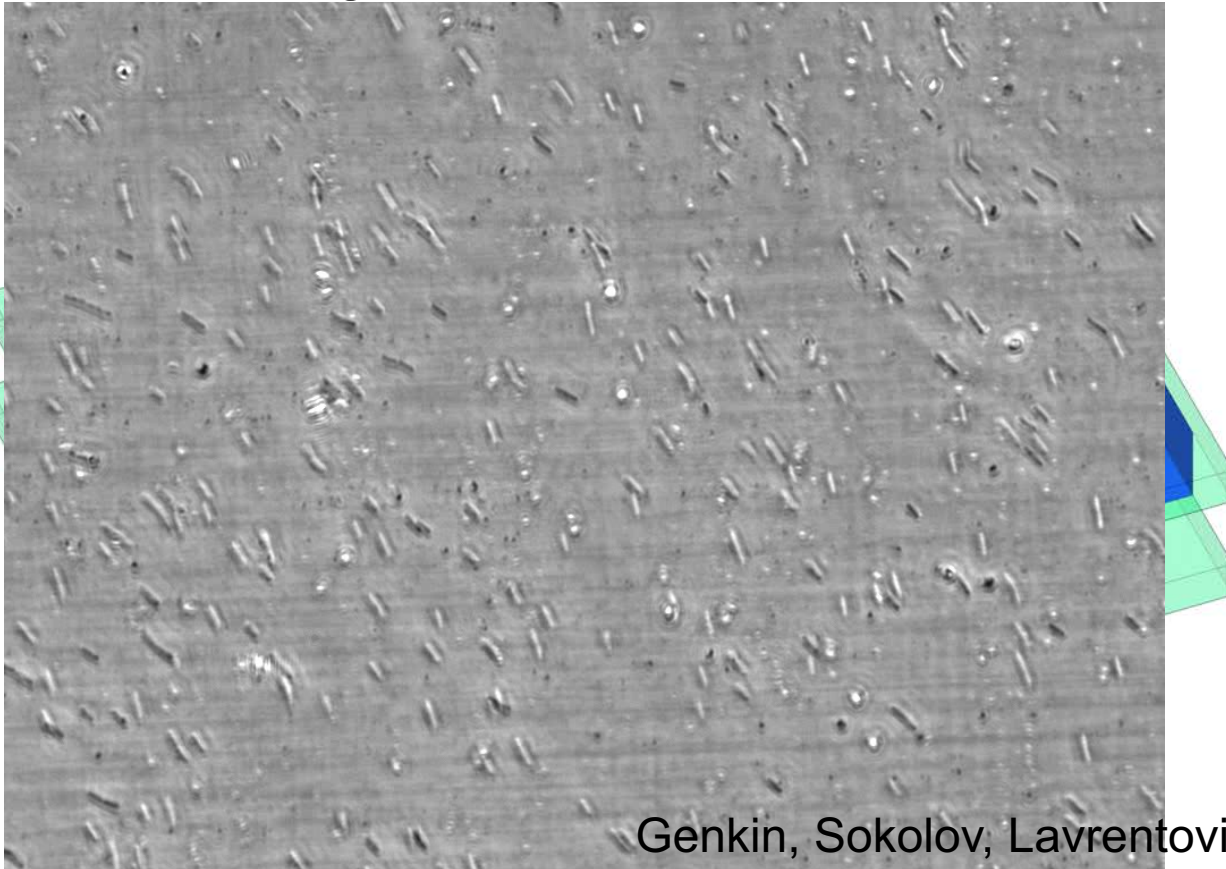
# Comparison with the Experiment

$$\xi = \sqrt{\frac{Kh}{\alpha c U_0}}$$

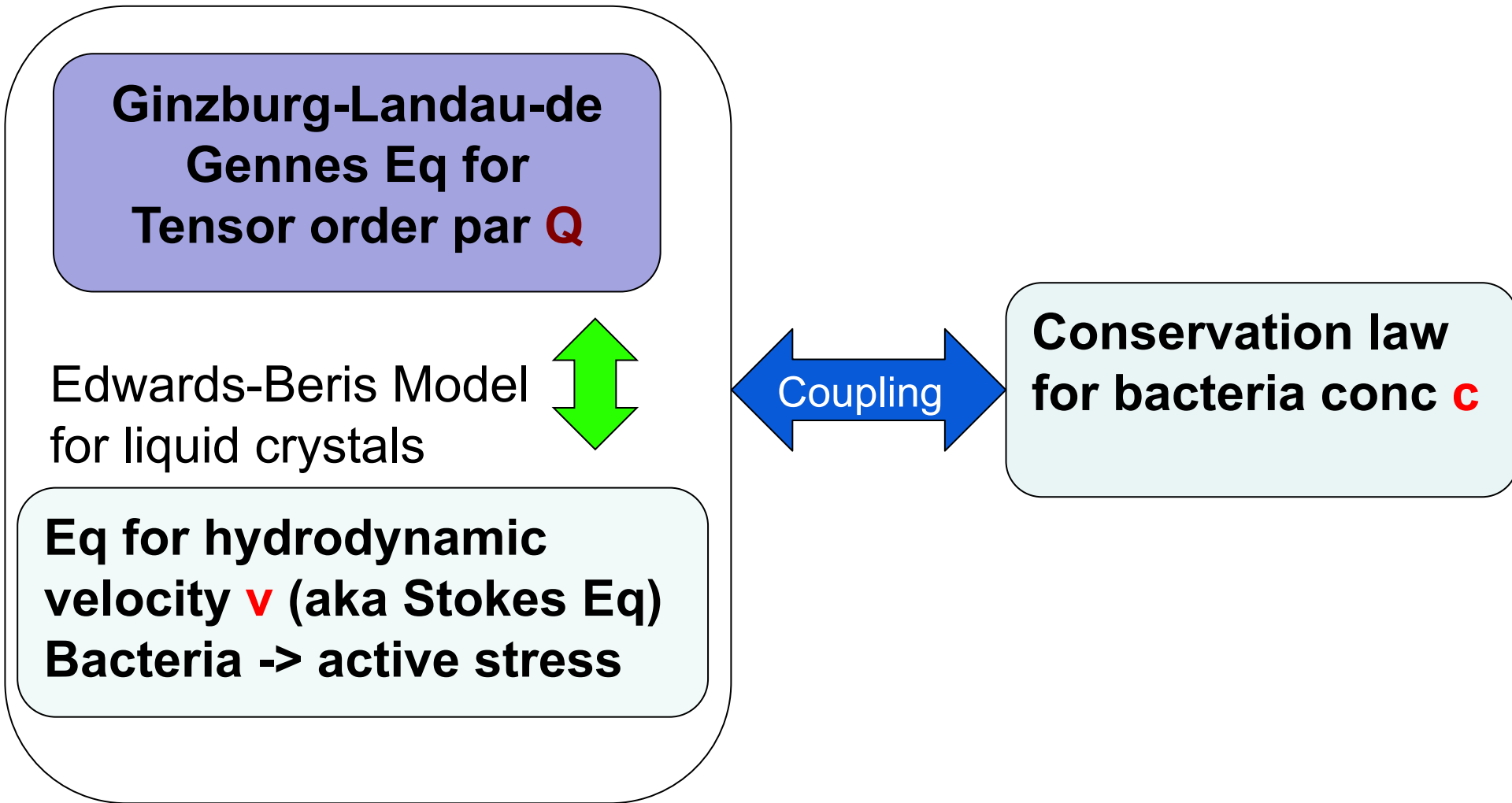


# Computational model

- Edwards-Beris model for liquid crystals (tensorial order par)
- thin layer approximation (2D description)
- almost no interaction between the bacteria
- bacteria impose stress on the fluid
- bacteria swim along the LC director



# Block-diagram of the model: 3 coupled PDE modules



# Edwards-Beris Model (LC)

$$(\partial_t + \mathbf{v}\nabla)\mathbf{Q} - \mathbf{S} - \Gamma\mathbf{H} + \mathbf{F}_{\text{exter}} = 0$$

2D order parameter (OP)  
Traceless anti-symmetric tensor

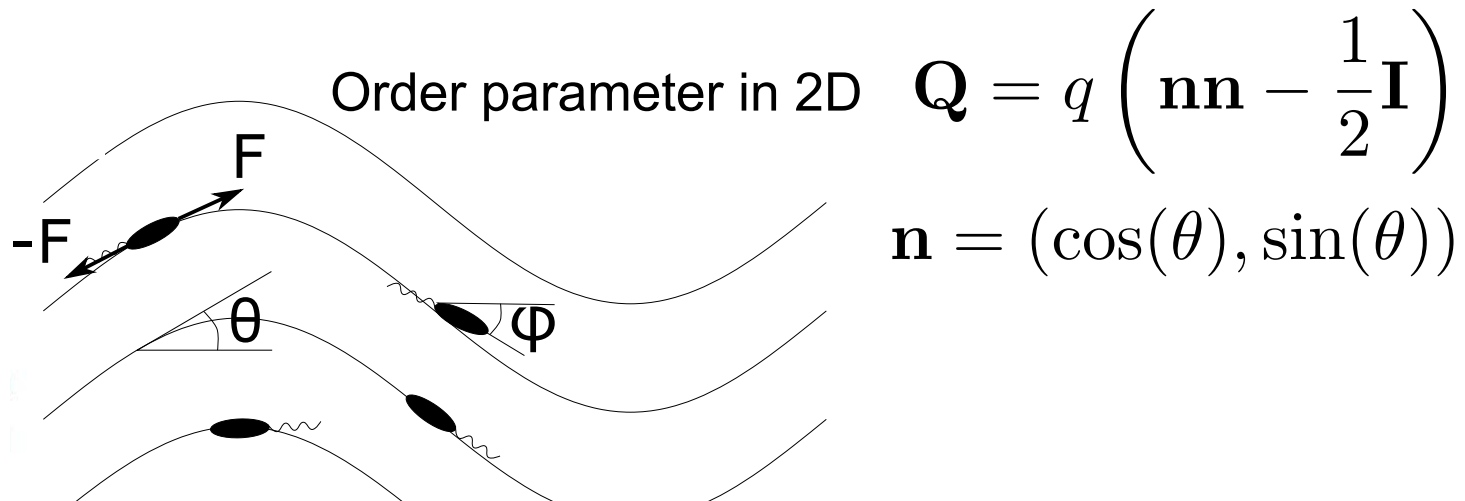
$$\mathbf{Q} = \begin{pmatrix} Q_{xx} & Q_{xy} \\ Q_{xy} & -Q_{xx} \end{pmatrix}$$

$\mathbf{v}$  – fluid flow

$\mathbf{H}$  – molecular field,  $\mathbf{S}$ -tensor

$\mathbf{F}_{\text{exter}}$  – external aligning field (anchoring)

$\Gamma$  – rate constant



# Edwards-Beris Model

**S** couples **Q** to the velocity gradient tensor **W**

Molecular field  $\mathbf{H} = -\frac{\delta F}{\delta \mathbf{Q}} - \frac{\mathbf{I}}{2} \text{Tr} \frac{\delta F}{\delta \mathbf{Q}}$

Ginzburg-Landau-de Gennes free energy  
(single constant approximation for elastic energy)

$$F = \int d\mathbf{r} \left( \frac{a}{2} Q_{\alpha\beta} Q_{\alpha\beta} - \frac{b}{3} Q_{\alpha\beta} Q_{\beta\gamma} Q_{\gamma\alpha} + \frac{c}{4} (Q_{\alpha\beta} Q_{\beta\alpha})^2 + \frac{L_1}{2} (\partial_\gamma Q_{\alpha\beta})^2 + \dots \right)$$

# Linear momentum equation (coupled to order par equation, generalization of Stokes eq)

$$\nabla \cdot \left( \overset{\text{symmetric}}{\downarrow} \sigma_a + \overset{\text{anti-symm}}{\downarrow} \sigma_s + \overset{\text{active (bact)}}{\downarrow} \sigma_{\text{act}} + \overset{\text{viscous}}{\downarrow} \sigma_{\text{visc}} - \overset{\text{pressure}}{\downarrow} p \mathbf{I} \right) - \overset{\text{wall friction}}{\downarrow} \xi \mathbf{v} = 0$$

$\mathbf{v}$  – hydrodynamic velocity

stress tensors  $\sigma$  are functions of  $\mathbf{Q}, \mathbf{v}$

$\xi$  - friction coefficient

Bacteria impose active stress  $\sigma_{\text{act}}$ ,  
drive the system out of equilibrium

# Active stress due to bacteria swimming

$$\sigma_{\text{act}} = \lambda c \left( \mathbf{p}\mathbf{p} - \frac{\mathbf{I}}{2} \right)$$

$$\mathbf{p} = (\cos(\phi), \sin(\phi))$$

$\mathbf{p}$  – orientation of bacteria

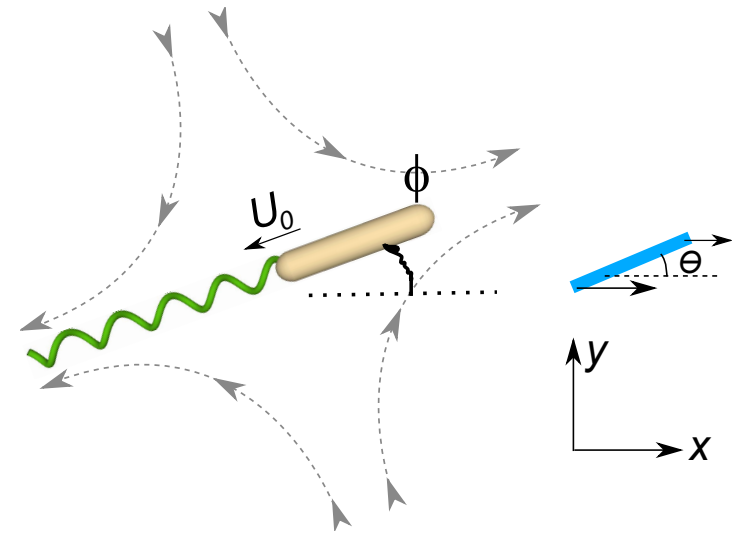
$\lambda < 0$  – stresslet strength (pusher)

$c$  – concentration of bacteria

$\phi$  – orientation of bacteria

$\theta$  – orientation of a nematic

$$\partial_t \phi = \frac{q}{\tau_0} \sin(2\theta - 2\phi) + \dots$$



$\tau_0 \ll 1$  – fast relaxation towards nematic direction

$\phi = \theta, \theta + \pi \rightarrow \mathbf{p} = \mathbf{n}, \mathbf{p} = -\mathbf{n}$

# Conservation law: transport of bacteria

- bacteria swim parallel/antiparallel to the director:  
 $\phi = \theta, \phi = \theta + \pi$  or  $\mathbf{p} = \mathbf{n}, \mathbf{p} = -\mathbf{n}$
- bacteria randomly reverse direction with the rate  $1/\tau$
- $c_+$  parallel to  $\mathbf{n}$ ,  $c_-$  antiparallel:  $-\pi/2 < \theta < \pi/2$

self-propulsion

advection

reversals

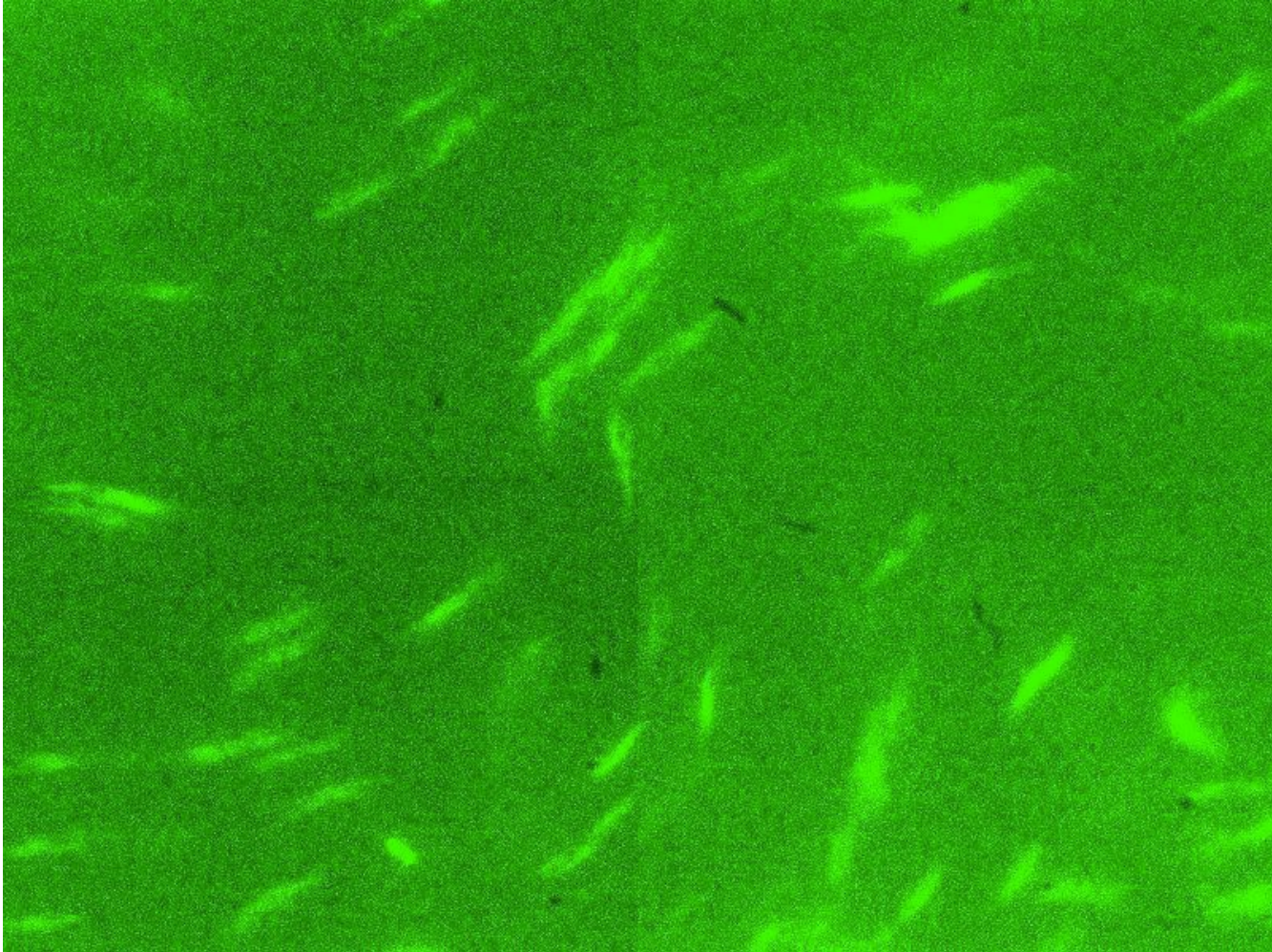
diffusion

$$\begin{aligned}
 \partial_t c_+ + \nabla \cdot (V_0 \mathbf{n} c_+ + \mathbf{v} c_+) &= -\frac{c_+ - c_-}{\tau} + D_c \nabla^2 c_+ \\
 \partial_t c_- + \nabla \cdot (-V_0 \mathbf{n} c_- + \mathbf{v} c_-) &= -\frac{c_- - c_+}{\tau} + D_c \nabla^2 c_- \\
 c &= c_+ + c_-
 \end{aligned}$$

$V_0$ - swimming speed



# The mechanism of reversal: two flagella bundles



Nuris Figueroa , PSU

# Relation to Active Nematic Model

- Amin Doostmohammadi, Julia Yeomans, others
- Equation for the OP

$$(\partial_t + \mathbf{v}\nabla)\mathbf{Q} - \mathbf{S} - \Gamma\mathbf{H} + \mathbf{F}_{\text{exter}} = 0$$

- Linear momentum

$$\nabla (\sigma_a + \sigma_s + \sigma_{\text{act}} + \sigma_{\text{visc}} - p\mathbf{I}) - \xi\mathbf{v} = 0$$

- Constant density. These eqs can be obtained in the limit of very small reversal time (unphysical limit)

# Analytical linear stability of the aligned state in bacteria-nematic system

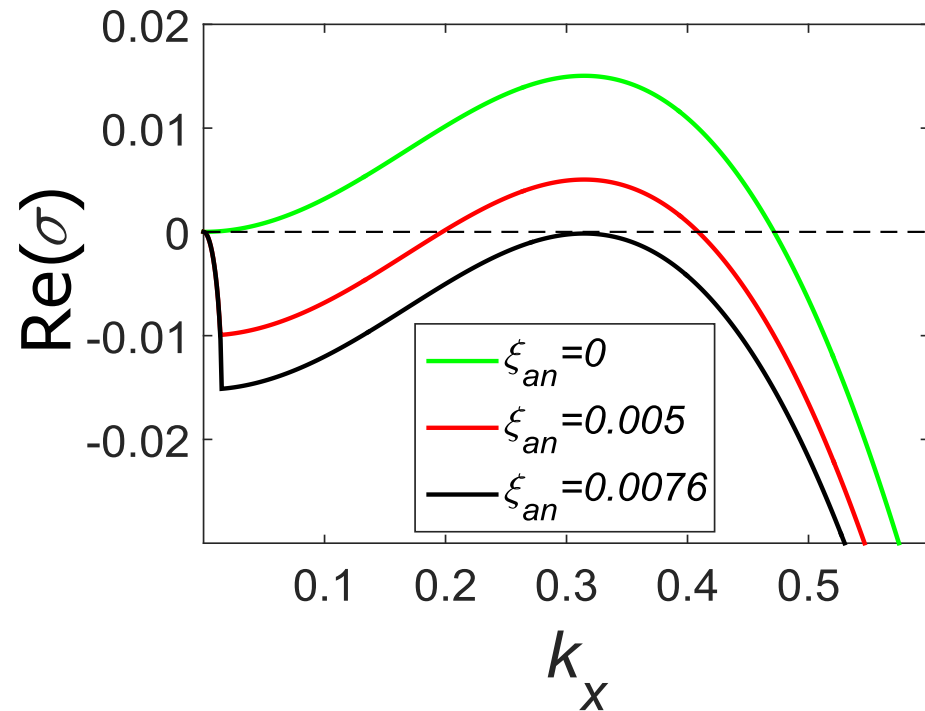
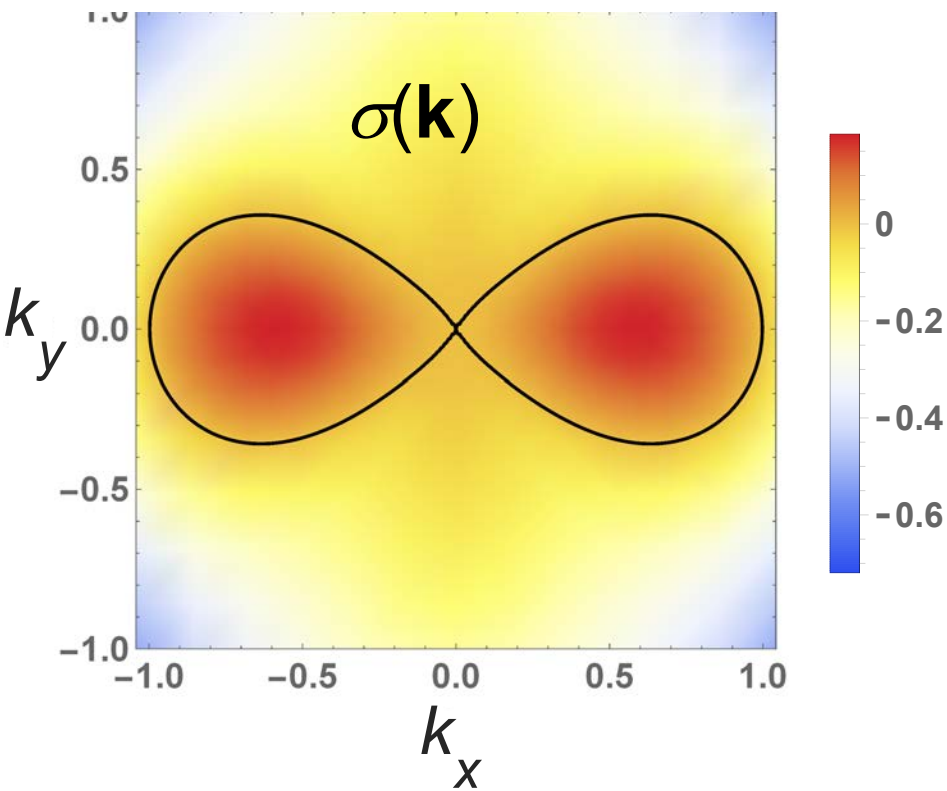
homogenous steady state:  $\mathbf{n} = (1, 0)$ ,  $c_+ = c_- = c_0/2$

perturbations  $\sim \exp(\sigma(\mathbf{k})t + i\mathbf{k}\mathbf{r})$

long-wave instability:  
zero anchoring

short-wave instability:  
nonzero anchoring

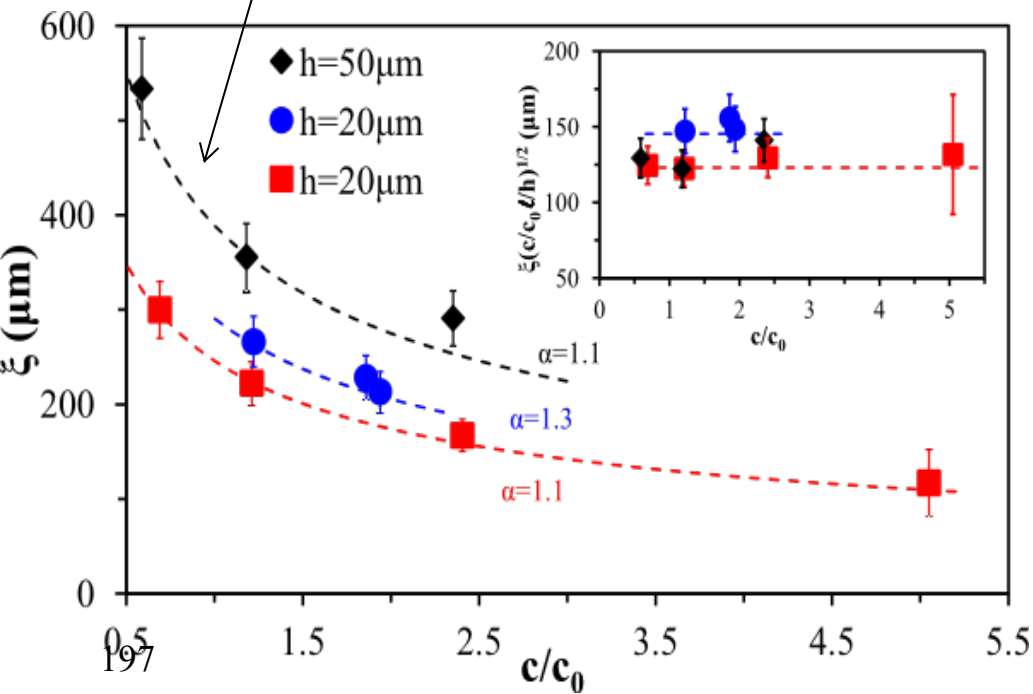
$$k_{\text{cr}} \sim (Er \xi_{\text{an}})^{1/4}$$



# Comparison with the Experiment

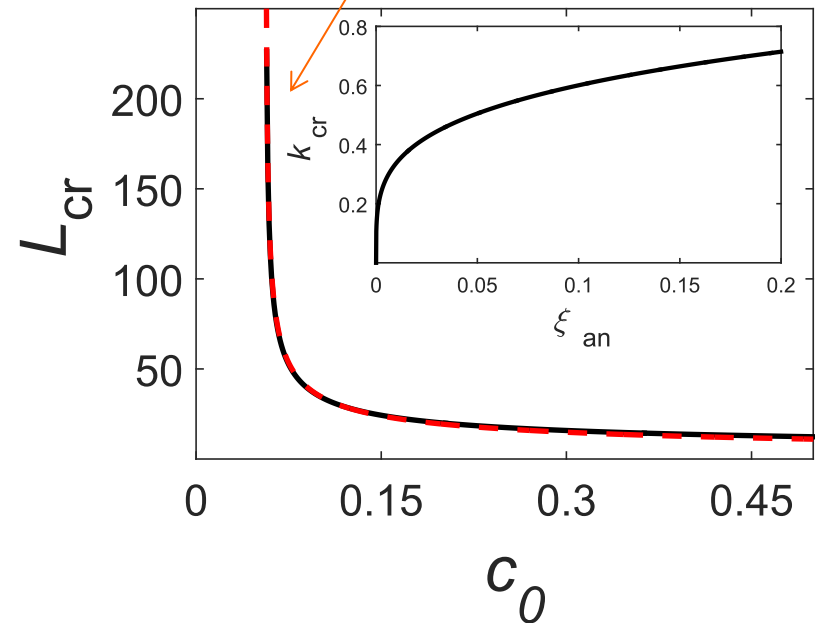
$$\xi = \sqrt{\frac{Kh}{\alpha c U_0}}$$

experiment



linear stability

$$L_{\text{cr}} \sim \frac{1}{\sqrt{c_0}}$$



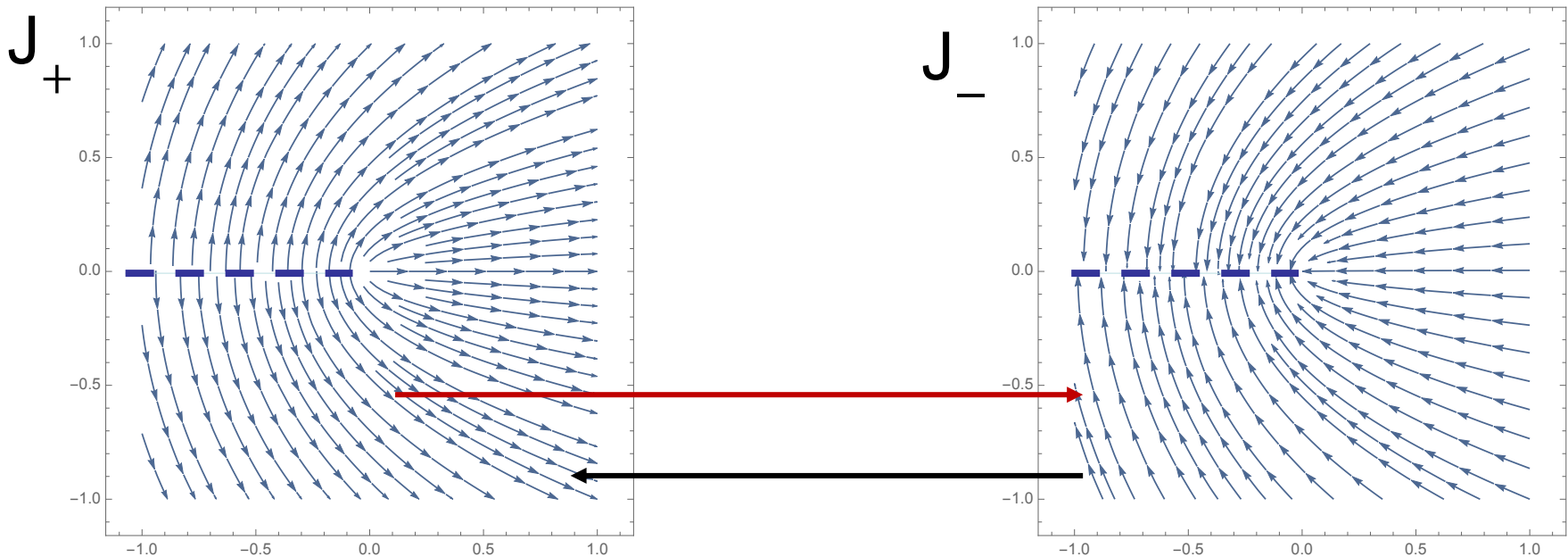
# Computational Analysis:

tensorial eqs coupled to conser law

- Semi-implicit quasi-spectral method (FFT)
- Massive parallel, implemented on the GPU
- # of mesh points  $1024^2$ , results checked for  $2048^2$
- Nvidia CUDA programming language
- Visualization – Matlab
- Custom-made defect tracking algorithm

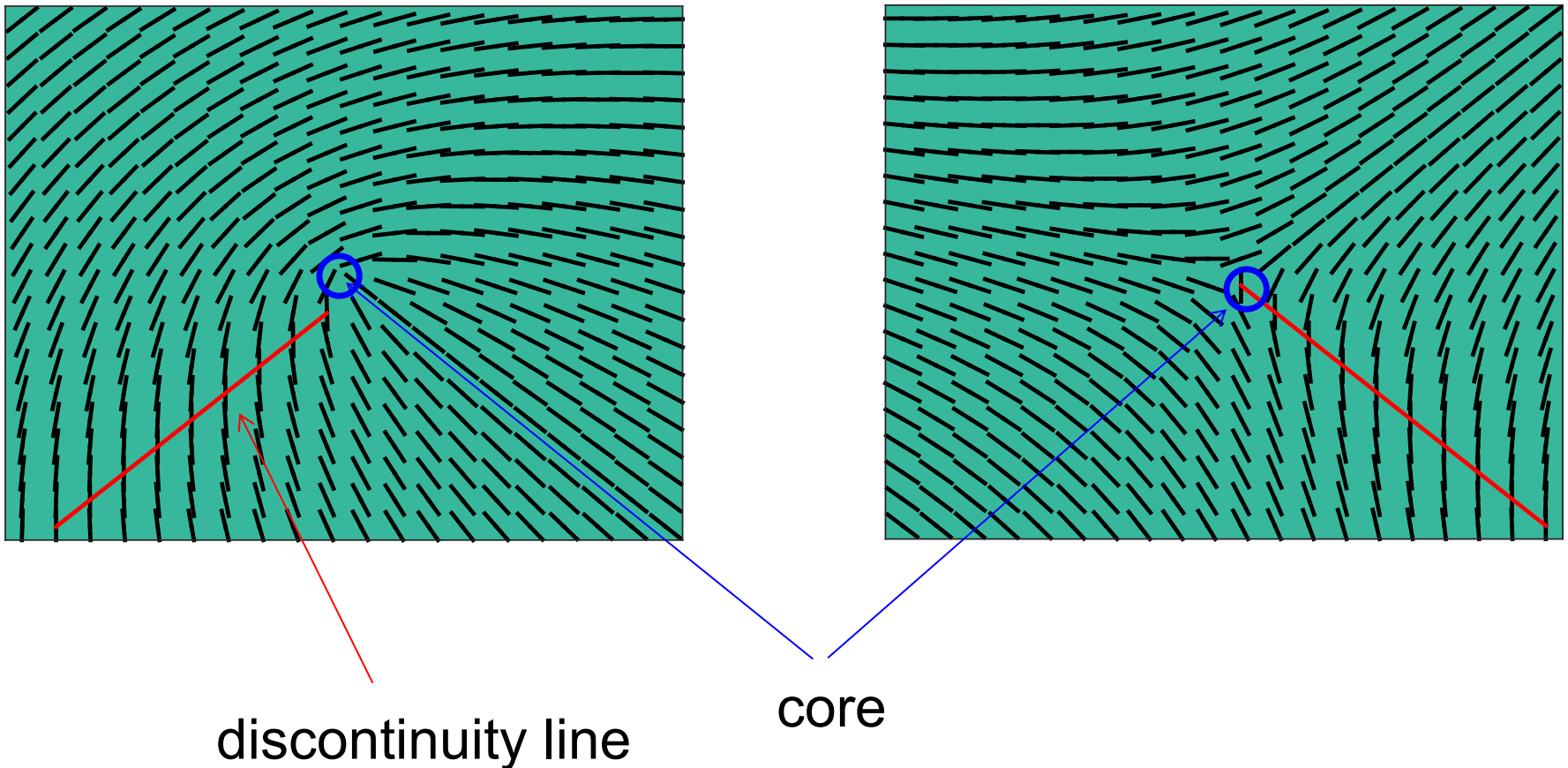
# Fundamental problem of algorithm

- Mapping a vector field (flux of bacteria) to a nematic field is ambiguous
- Nematic director  $\mathbf{n}=(\cos(\theta), \sin(\theta))$ , nematic angle  $-\pi/2 < \theta < \pi/2$
- Bacterial fluxes  $\mathbf{J}_{\pm} = \pm V_0 \mathbf{n}$  are discontinuous for  $\theta$  changing from  $-\pi/2$  to  $\pi/2$  : no standard numerical approach works



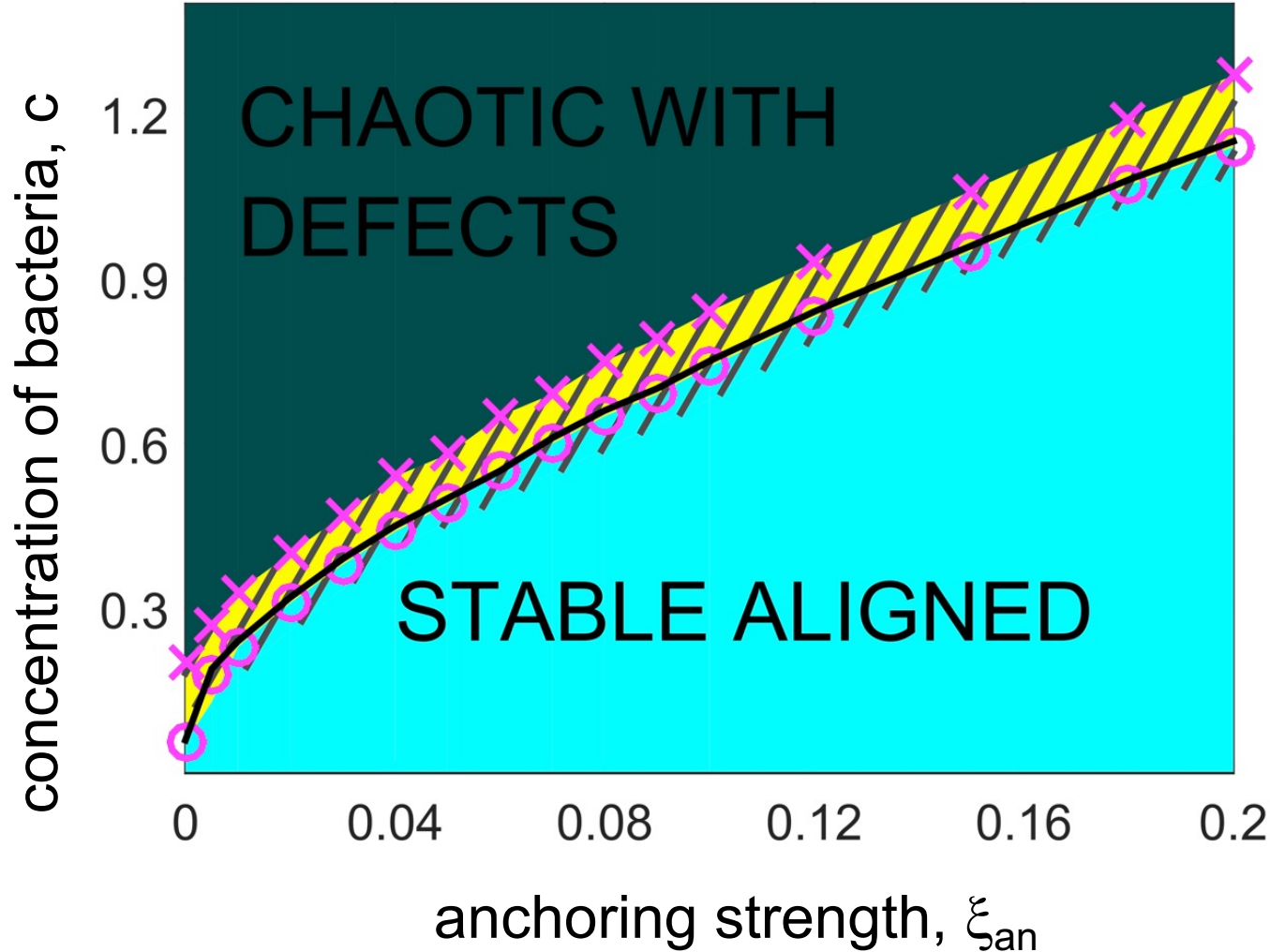
# resolving angle ambiguity

- nematic angle is defined between  $-\pi/2 < \theta < \pi/2$
- directions  $-\pi/2$  and  $\pi/2$  are identical for the nematic
- but the flux  $V_0 \sin(\theta)$  changes sign:  $\partial_t c \pm V_0 \nabla \cdot \mathbf{n} c + \dots$
- unphysical discontinuities for  $c_{\pm}$  along these lines



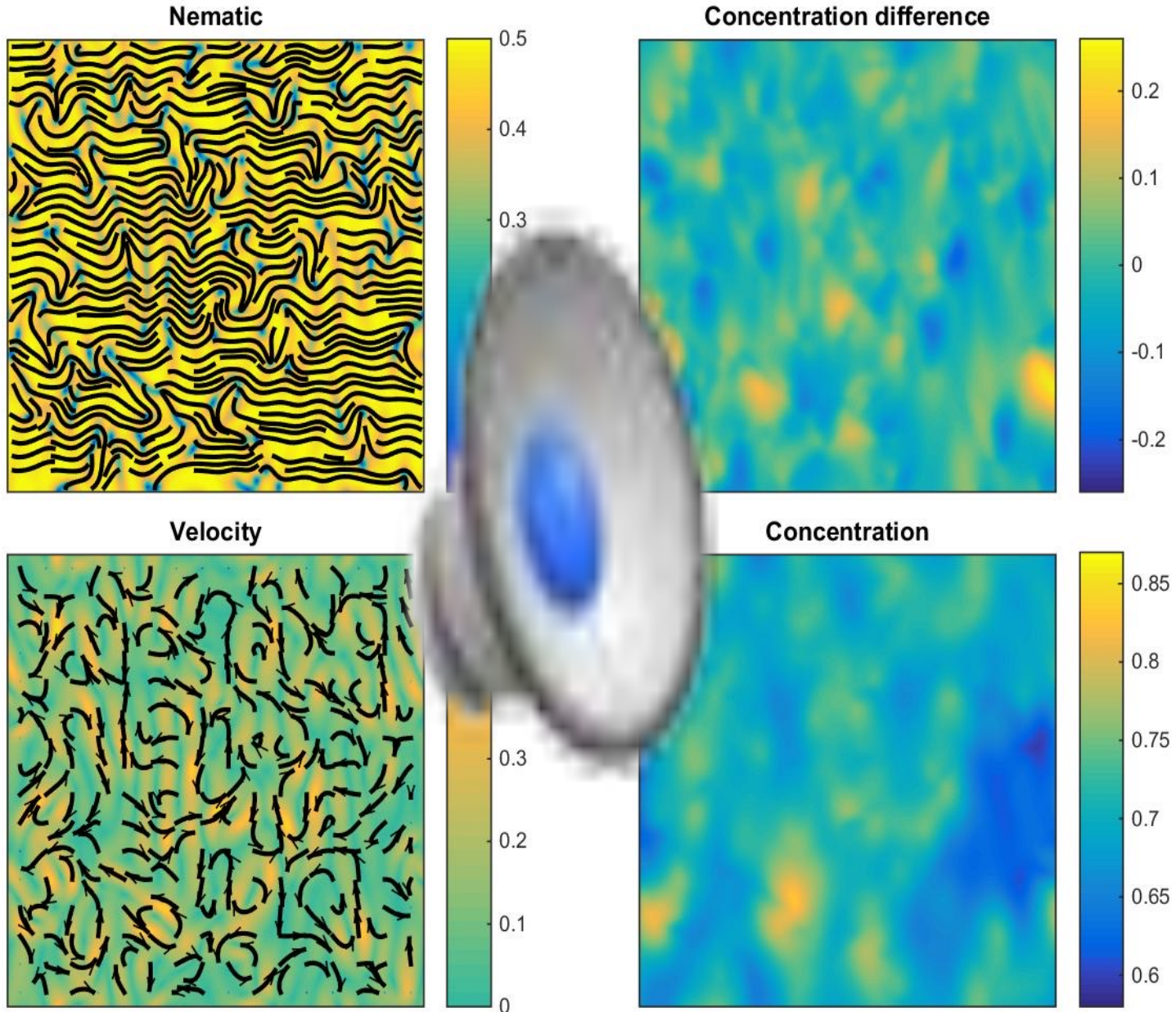
# Solution of PDEs - Phase diagram

- narrow band of periodic states
- hysteretic transition

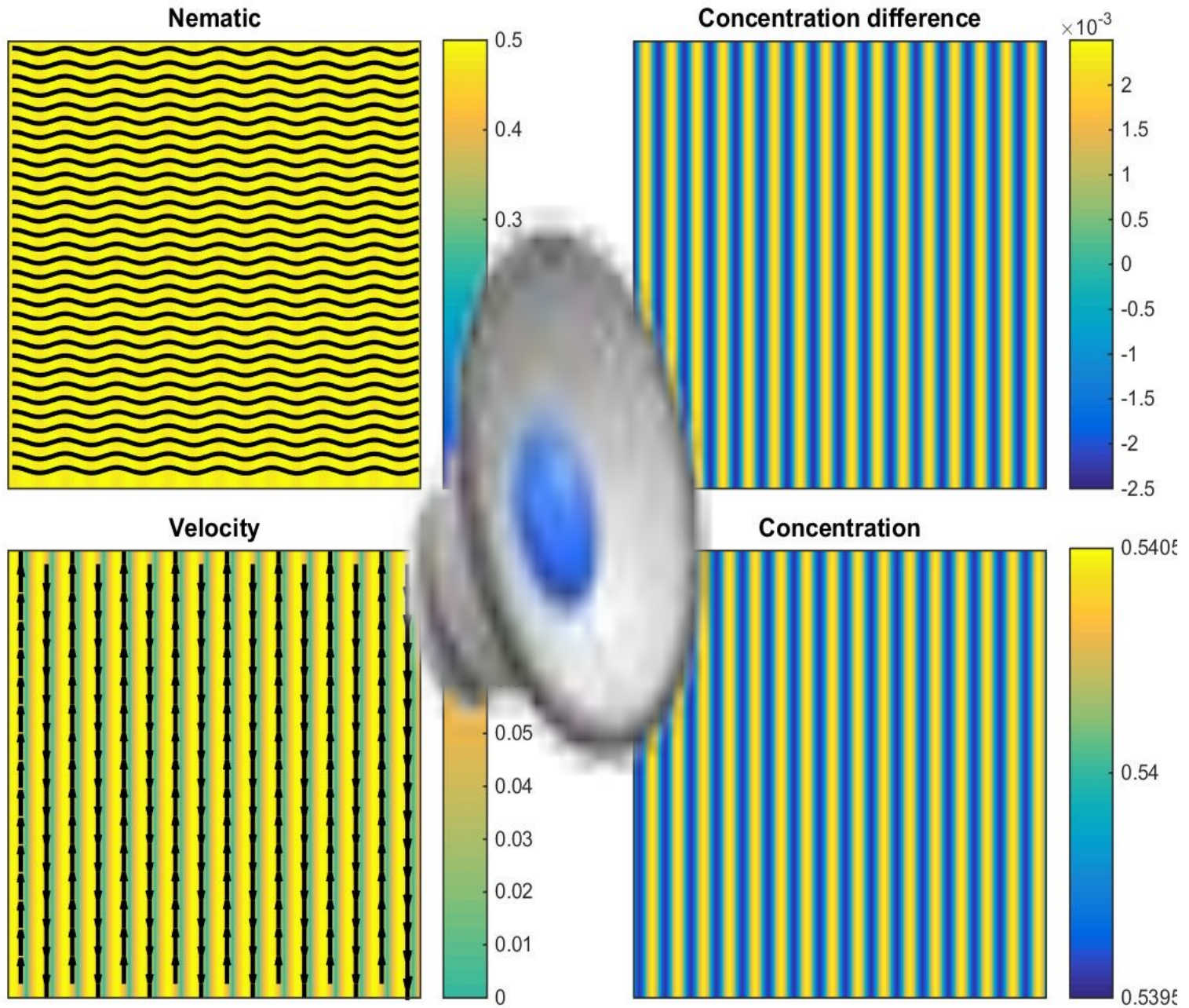




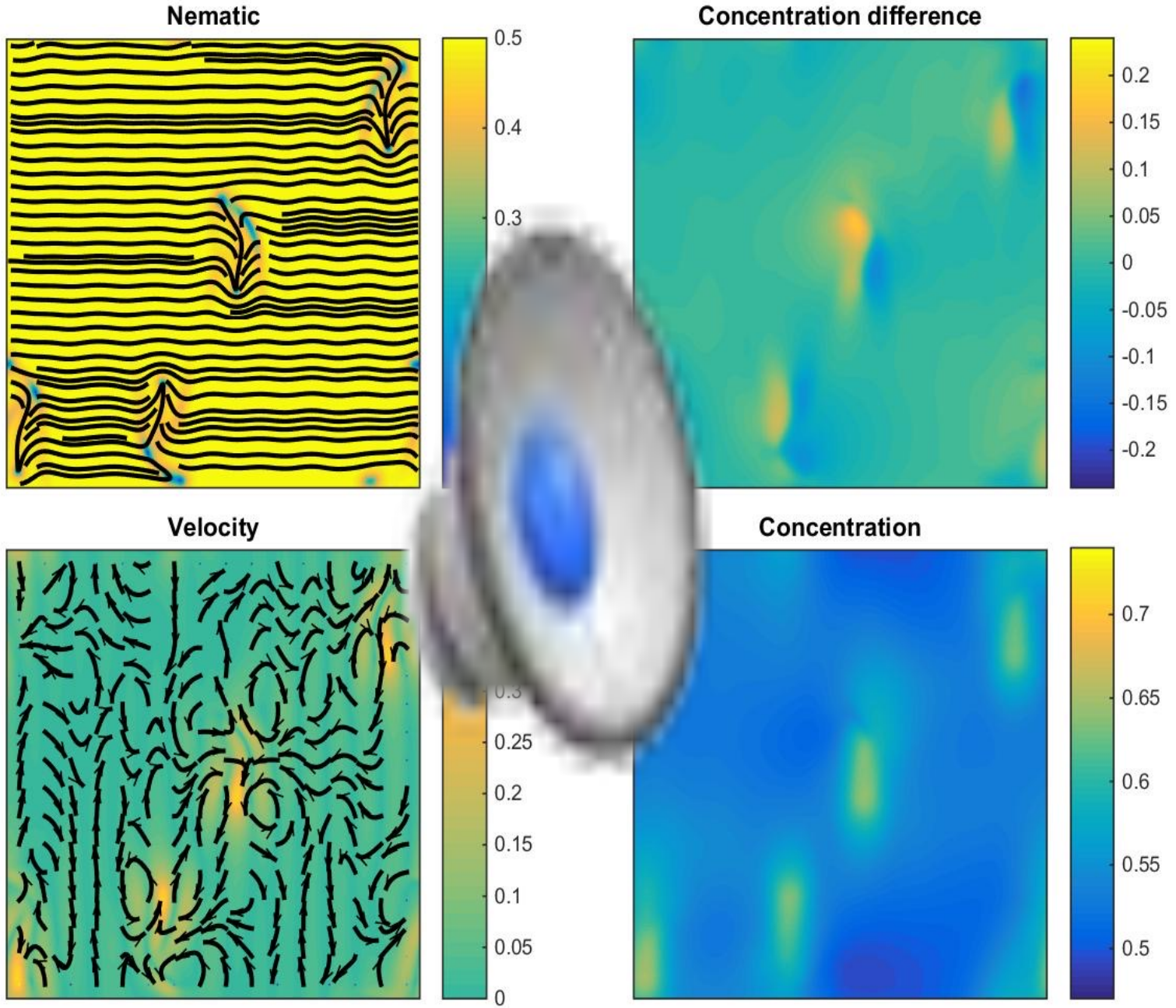
# instability, near threshold



# Anchoring, periodic modulations



# Strong anchoring, bound pairs

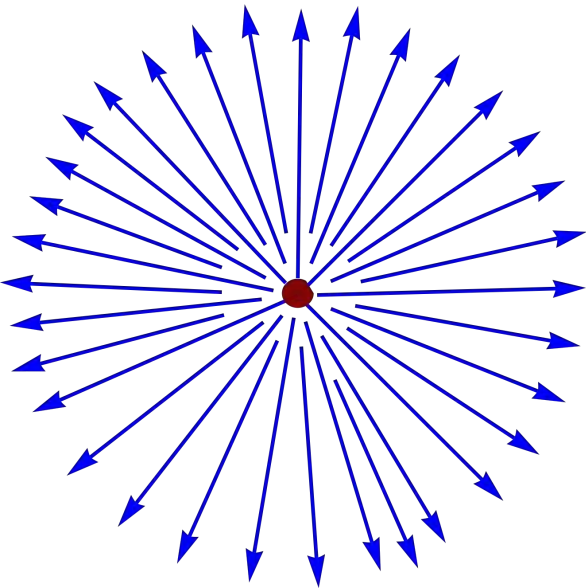


# Topological defects in a nematic

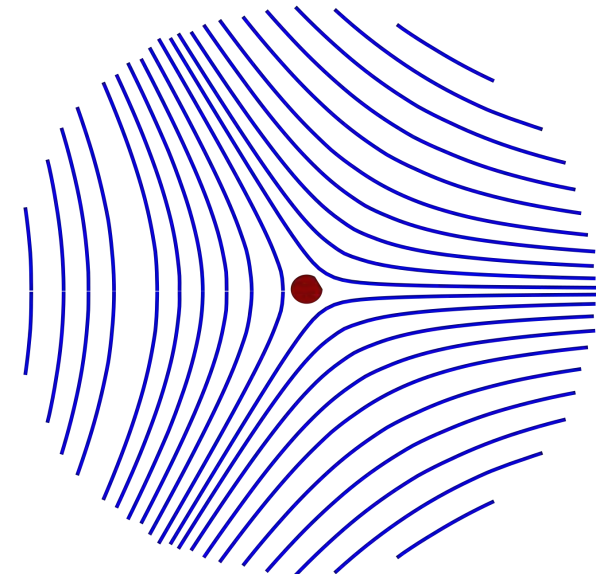
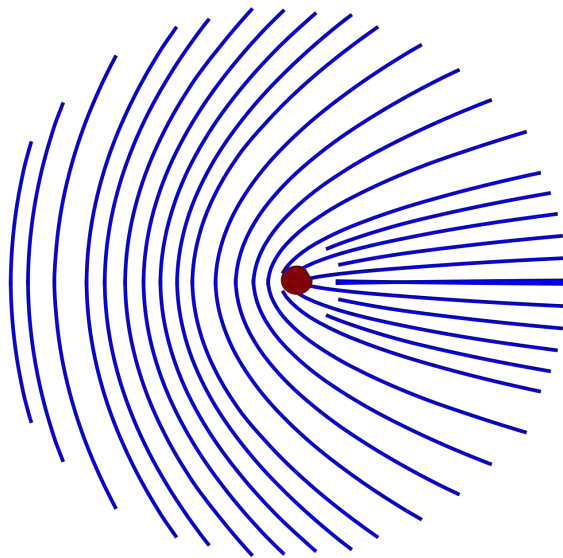
Topological charge  $k = \frac{1}{2\pi} \oint_{\gamma} \nabla \theta \cdot d\mathbf{l}$

$\theta$  orientation of director,  $\gamma$  contour

Integer defect  
(unstable)  
 $k=1$



Stable half-integer defects (disclinations)  
 $k=1/2$   $k=-1/2$

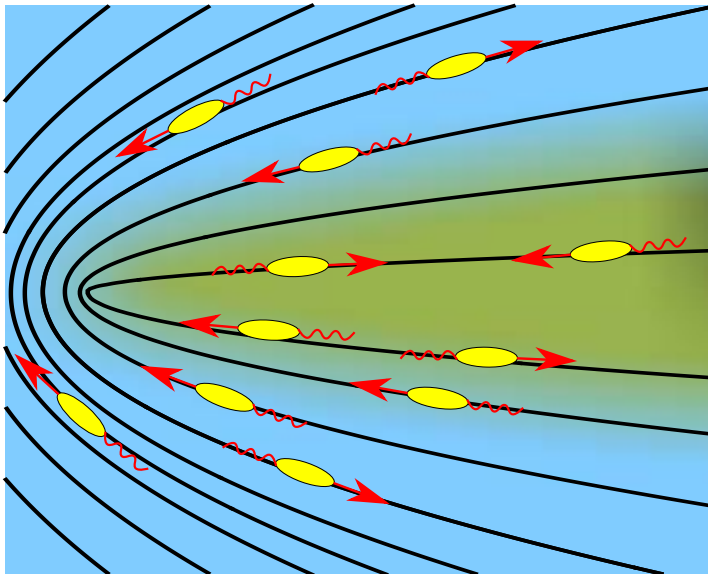


# passive versus active nematic

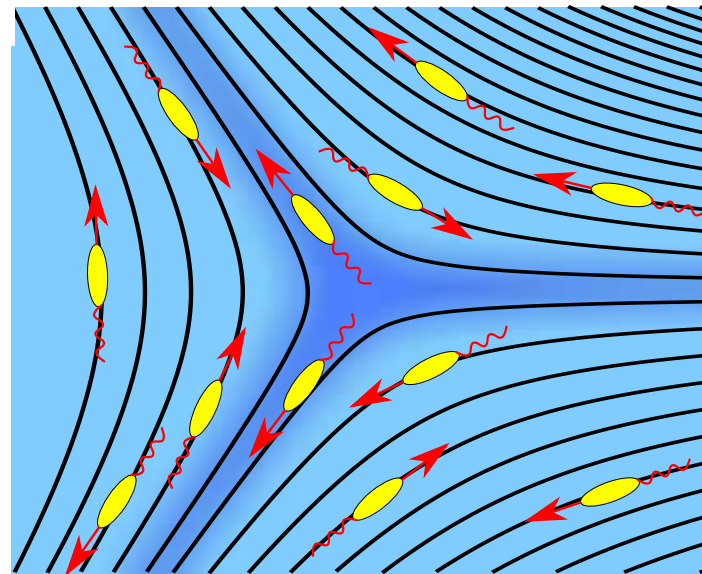
passive nematic: isolated topological defects are immobile

active nematic:  $-1/2$  defect is immobile (symmetry),  
but  $1/2$  defect moves spontaneously

$1/2$  defect: migration



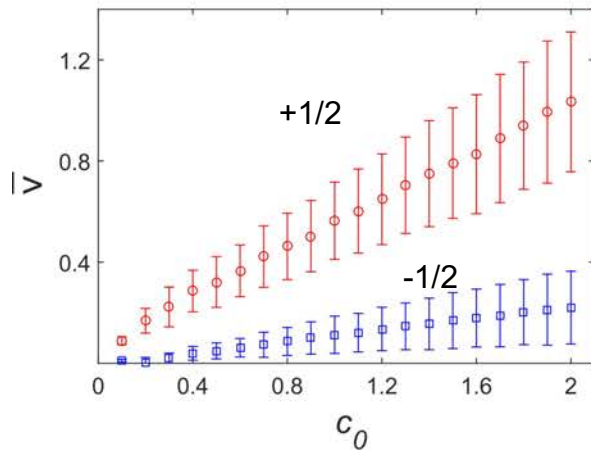
$-1/2$  defect: stationary



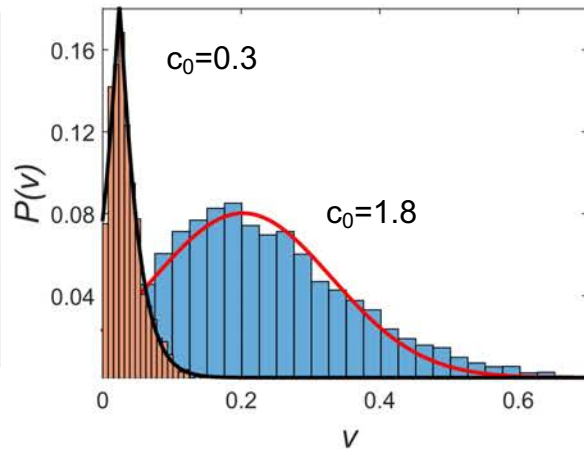
# Defect's statistics

- $+1/2$  defects persistently moving,  $-1/2$  defects are immobile
- Gaussian velocity distributions for high concentrations
- Stretched exponential for low concentrations ( $1 < \xi < 2$ )

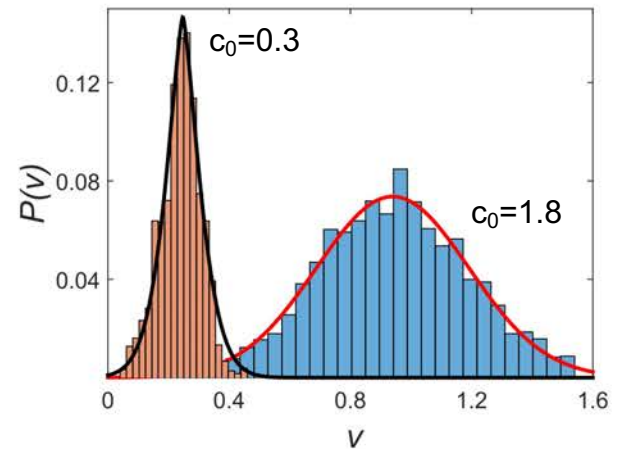
Defect's speed



$-1/2$  defects



$1/2$  defects

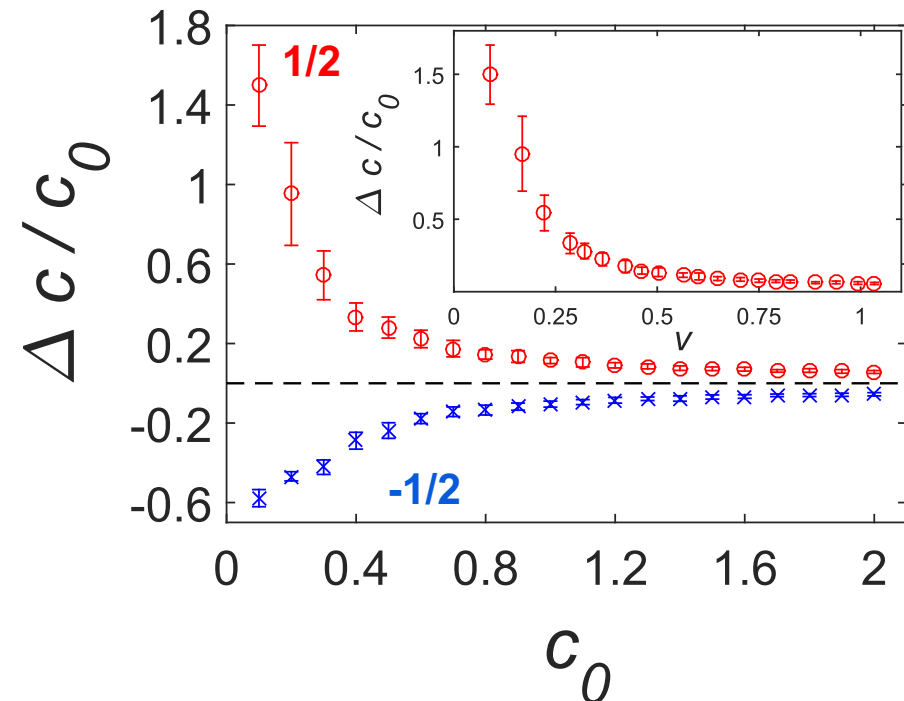
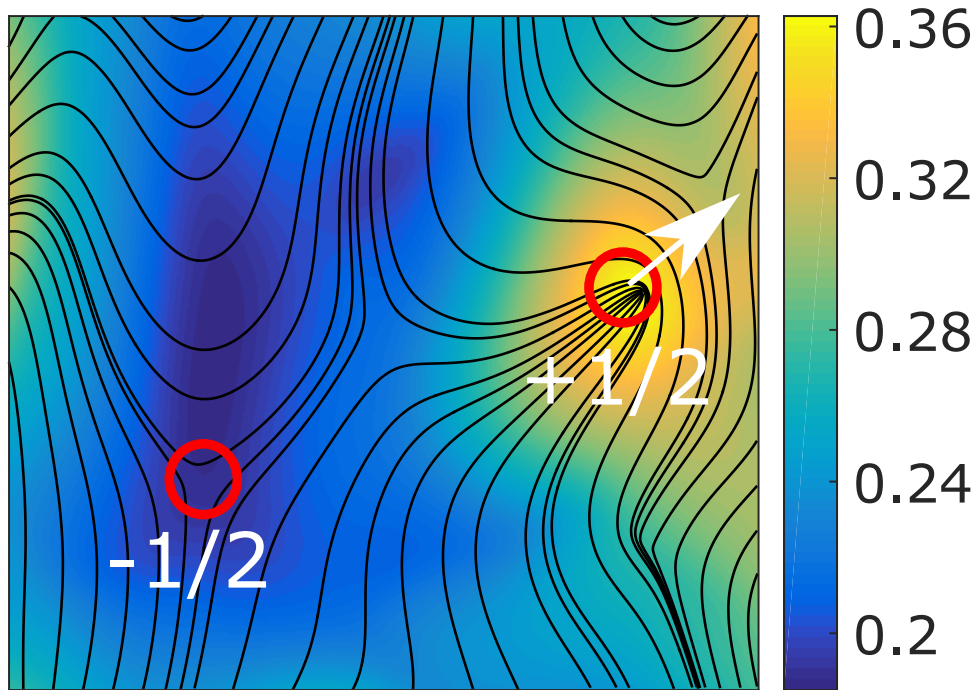


# Accumulation/depletion of bacteria in the cores of defects

## Relation between activity and topology

$\frac{1}{2}$  accumulation  
 $-\frac{1}{2}$  depletion

dependence on  
concentration

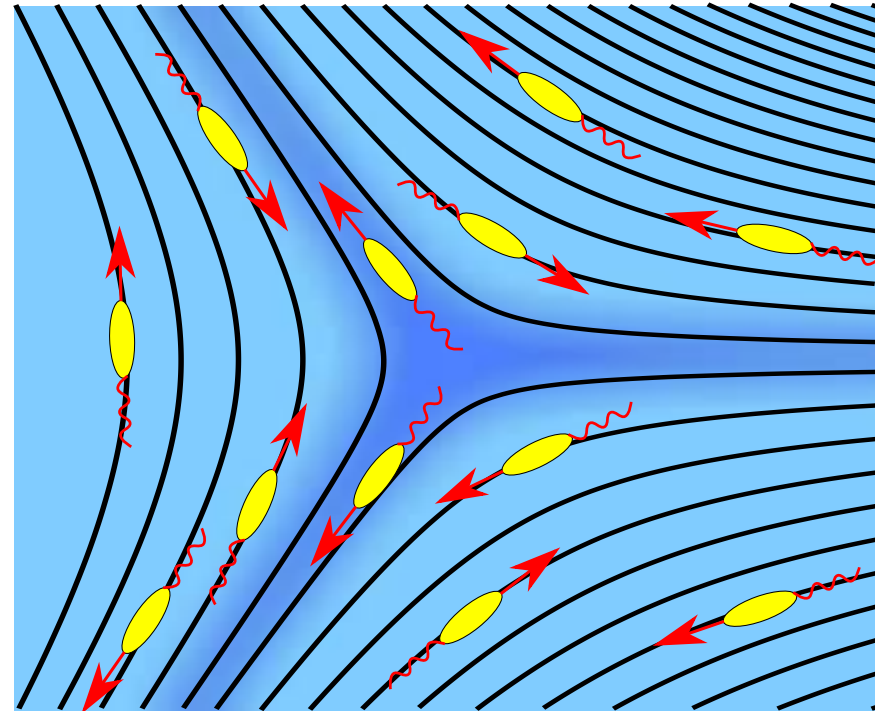


# Accumulation/depletion

large reversal time

$\frac{1}{2}$  defect accumulation

$-\frac{1}{2}$  defect depletion





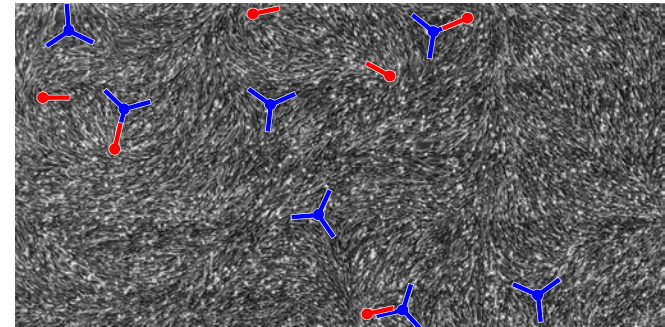
# Topological defects and tissue dynamics

LETTER

doi:10.1038/nature22321

## Topological defects control collective dynamics in neural progenitor cell cultures

Kyogo Kawaguchi<sup>1,2,3</sup>, Ryoichiro Kageyama<sup>4</sup> & Masaki Sano<sup>1,3</sup>

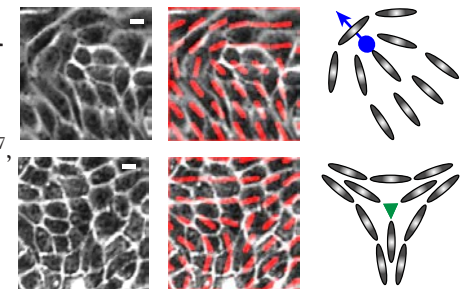


doi:10.1038/nature21718

LETTER

## Topological defects in epithelia govern cell death and extrusion

Thuan Beng Saw<sup>1,2\*</sup>, Amin Doostmohammadi<sup>3\*</sup>, Vincent Nier<sup>4</sup>, Leyla Kocgozlu<sup>1</sup>, Sumesh Thampi<sup>3,5</sup>, Yusuke Toyama<sup>1,6,7</sup>, Philippe Marcq<sup>4</sup>, Chwee Teck Lim<sup>1,2</sup>, Julia M. Yeomans<sup>3</sup> & Benoit Ladoux<sup>1,8</sup>



# Simplified conservation law

$$c_{\pm}(x, y, t) \rightarrow c_{\pm}(x - Vt, y)$$

$V$  - velocity of the defect

**Discontinues flux of bacteria  $V_0 \mathbf{n}$**

$\mathbf{n} = (\cos(\theta), \sin(\theta))$ ,  $\theta = \pm \varphi/2$  topological  $\pm 1/2$  defect

$\varphi$  - polar angle

$$\begin{array}{l} -V \partial_x c_+ \\ -V \partial_x c_- \end{array} \begin{array}{c} + \\ - \end{array} \boxed{V_0 \nabla \mathbf{n} c_{\pm}} = \begin{array}{l} -\frac{c_+ - c_-}{\tau} + D_c \nabla^2 c_+ \\ -\frac{c_- - c_+}{\tau} + D_c \nabla^2 c_- \end{array}$$

*Additional boundary condition along the branch cut  $\theta = \pm \pi/2$   
Regularization removes discontinuity*

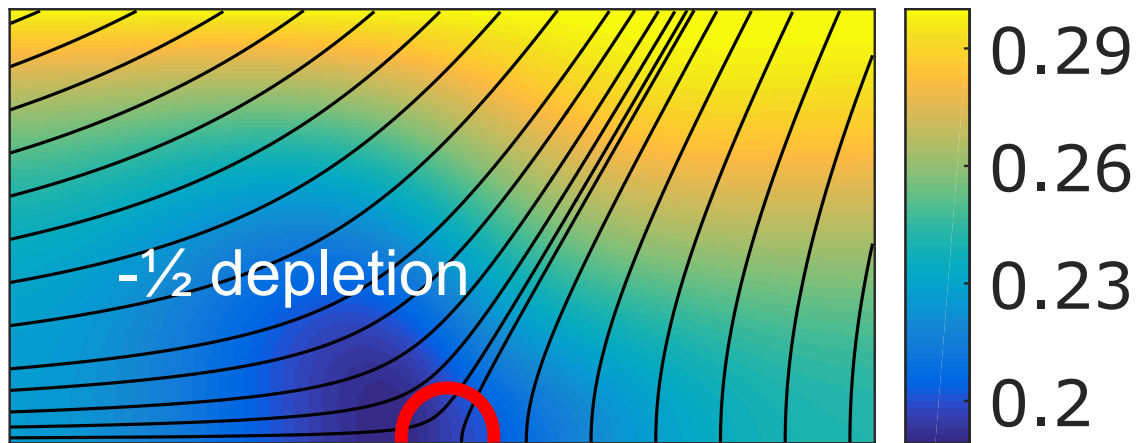
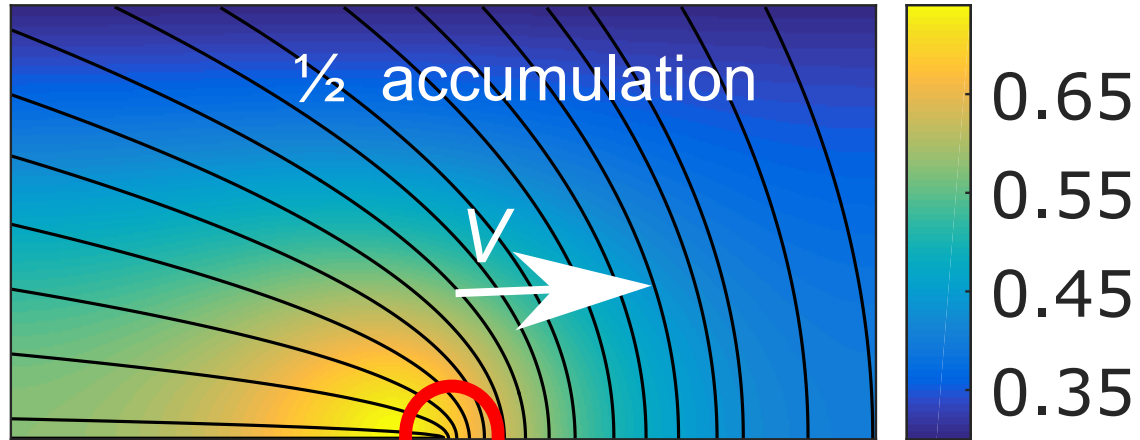
# Analytical solution for $\tau \rightarrow 0$ (small reversal time)

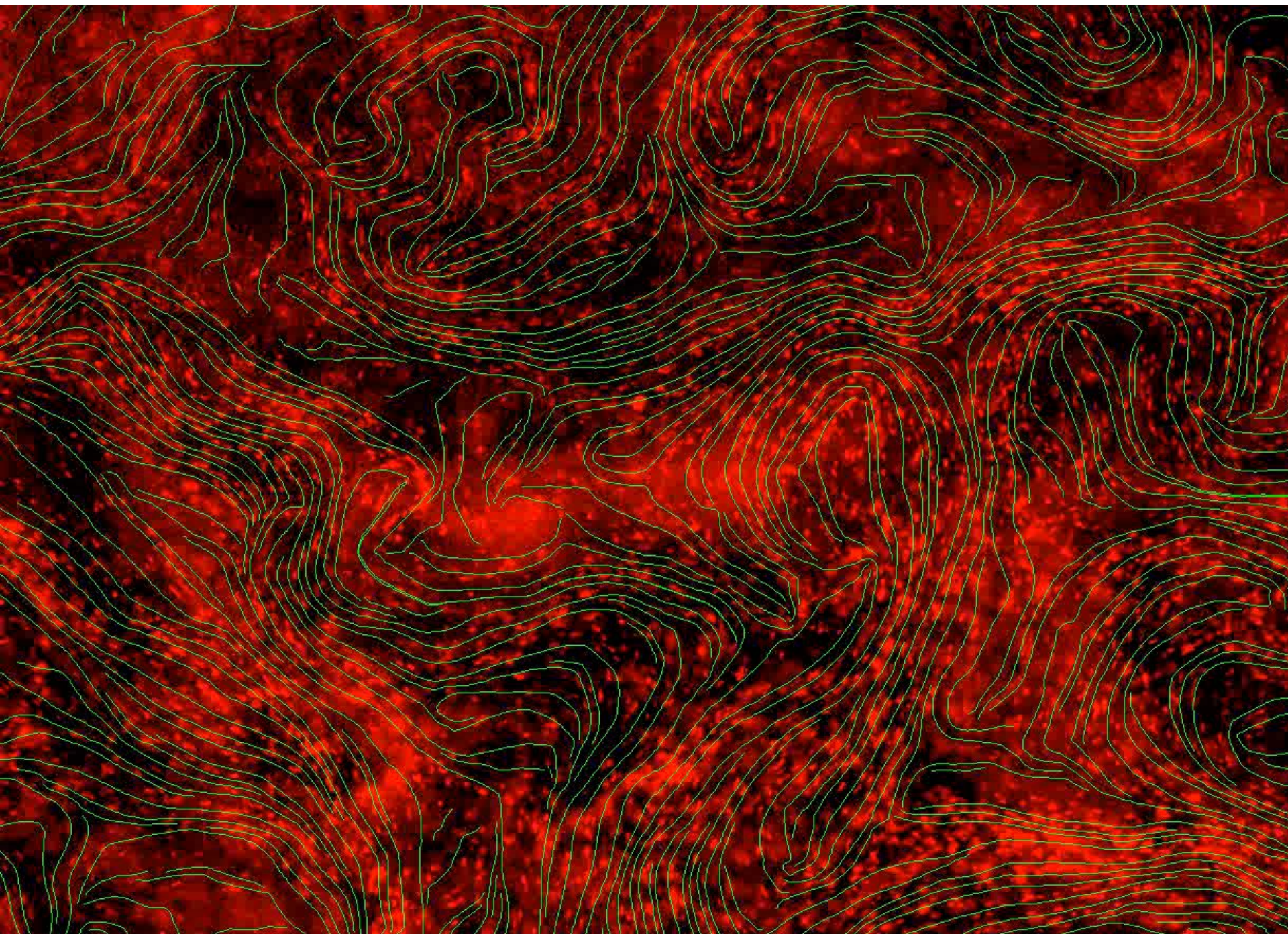
$$c_{1/2} \approx c_0 + \frac{\tau V_0^2 c_0}{8D_c} \cos \varphi$$

$$c_{-1/2} \approx c_0 - \frac{\tau V_0^2 c_0}{24D_c} \cos 3\varphi$$

no accumulation /depletion

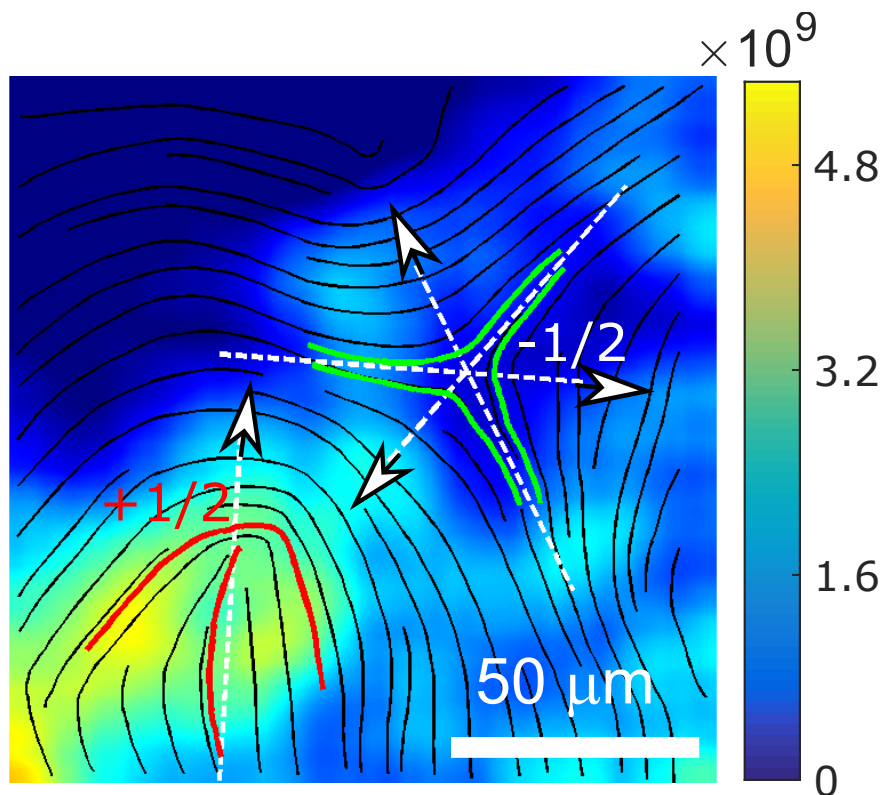
# Numerical solution (large reversal time)



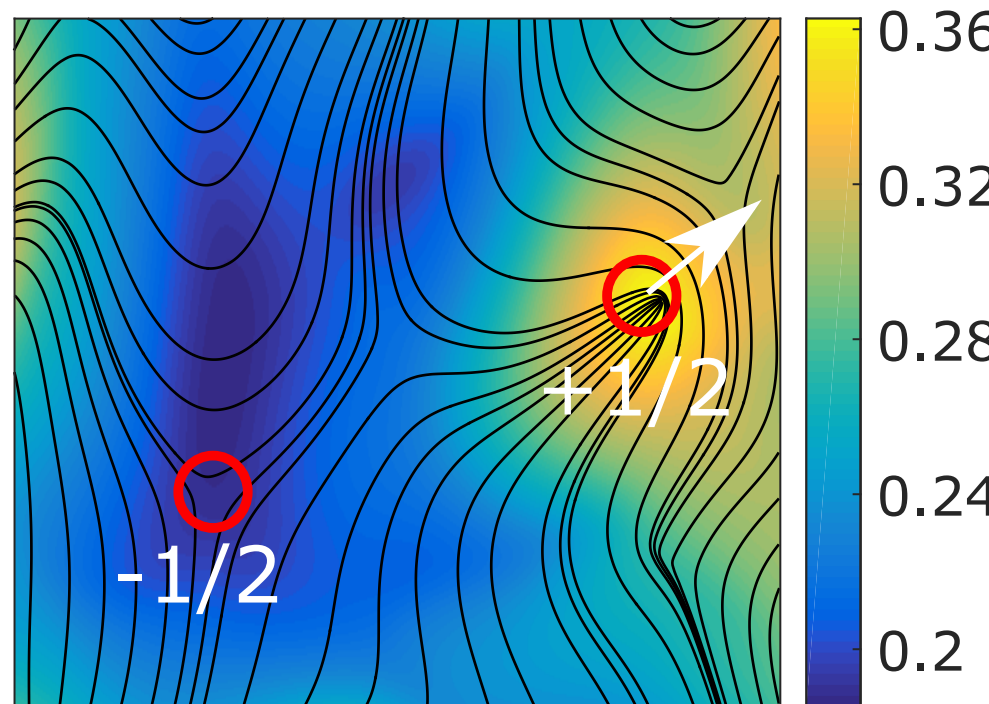


# Comparison of theory and experiment

experiment

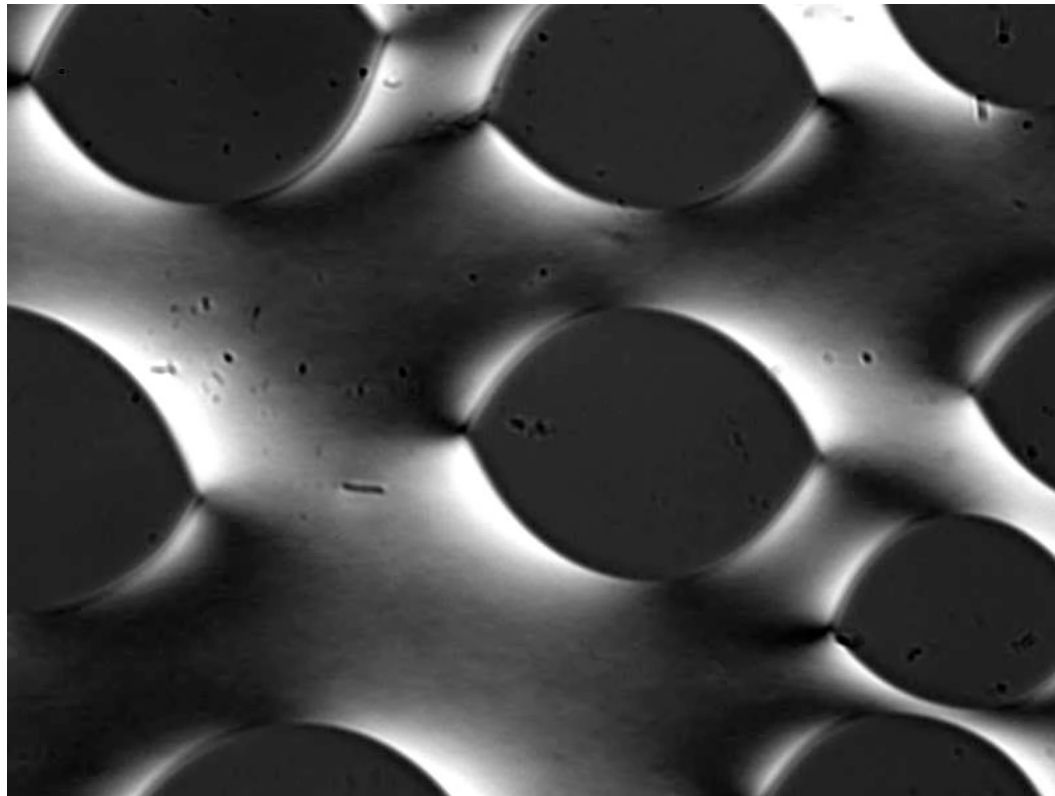


theory



# Tactoids and Turbulence

- Isotropic inclusions in nematic phase
- Shape, size, etc. is determined by interplay of tension on the I-N interface and elasticity of the nematic phase



# Numerical implementation of tactoids

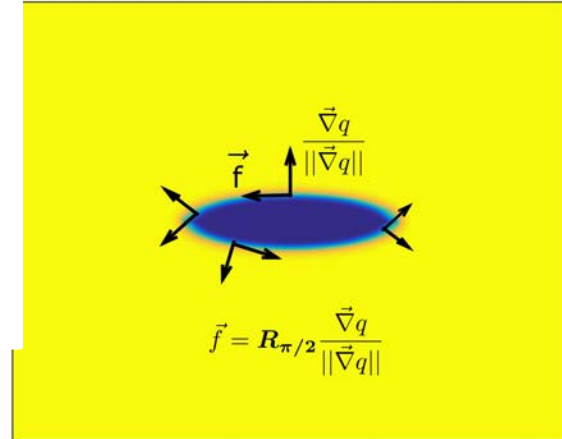
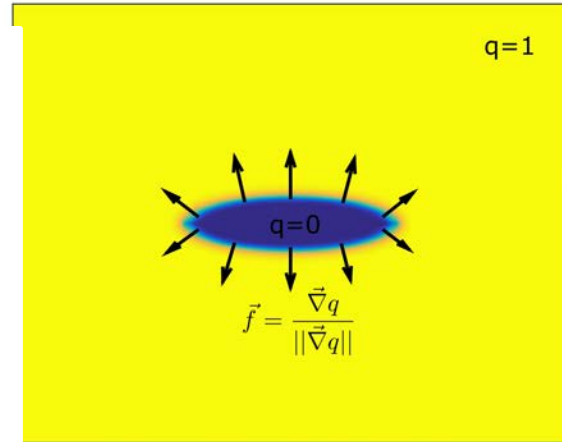
- Landau-de Gennes free energy:

$$\mathcal{F} = \int dr^3 \left( -\frac{a}{2} \mathbf{Q} : \mathbf{Q} + \frac{c}{4} (\mathbf{Q} : \mathbf{Q})^2 + \frac{K}{2} (\nabla \mathbf{Q} : \nabla \mathbf{Q}) \right)$$

- Equilibrium amplitude of the OP:

$$q = \begin{cases} \sqrt{\frac{a}{2c}}, & a > 0, c > 0 \\ q = 0, & a < 0, c > 0 \end{cases}$$

- Set  $q = 0$  in I phase,  $q \neq 0$  in N phase
- Set normal to the surface parallel to the gradient of the OP
- Tactoids shape is dynamically adjusted
- Planar and homeotropic anchoring





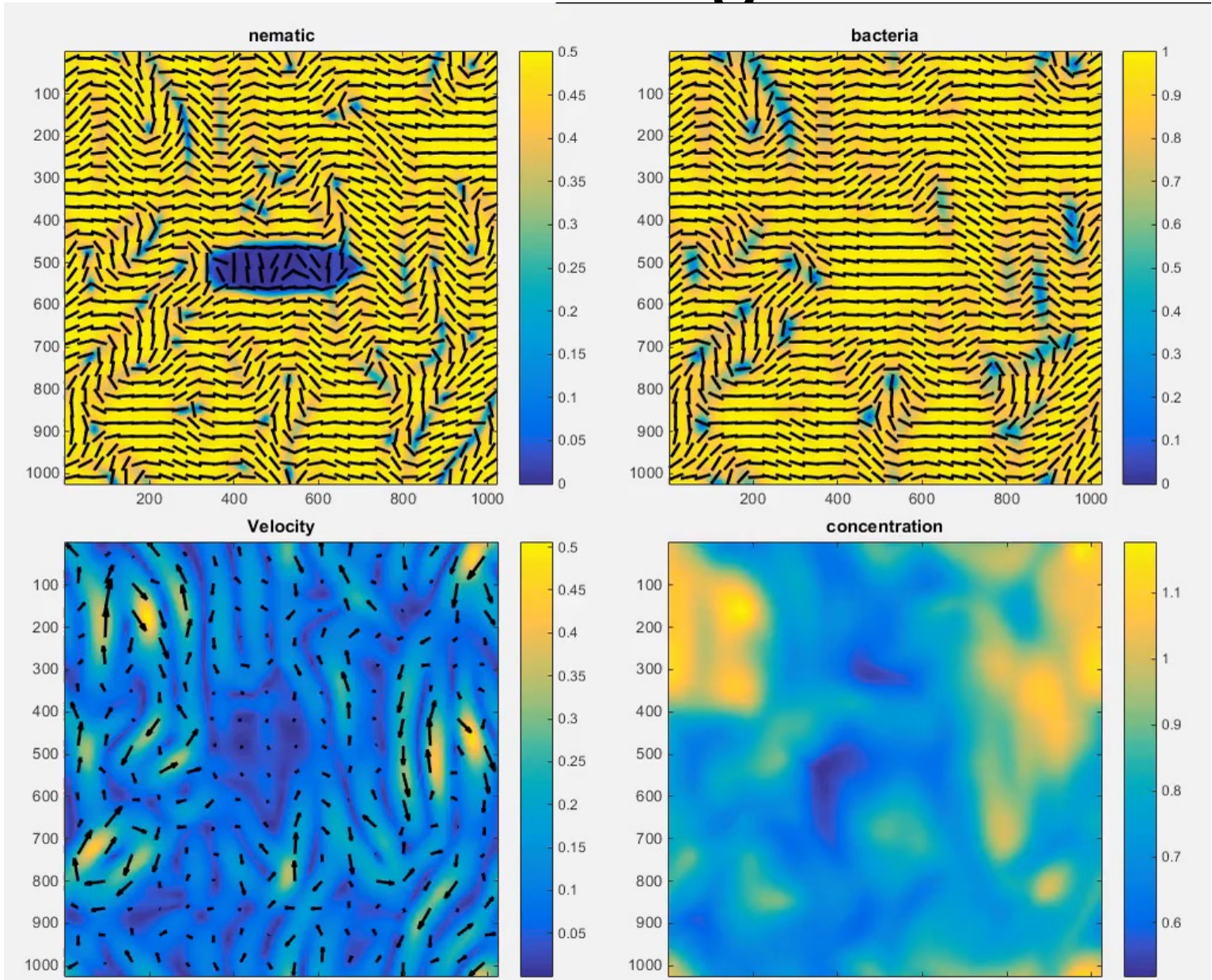
# Enforcing the alignment

- Typically, a planar alignment on the tactoid's surface
- Alignment enforced by the Rapini-Papolar condition (equivalent to Robin b.c.)

$$\sigma = w(\theta_0 - \theta)^2$$

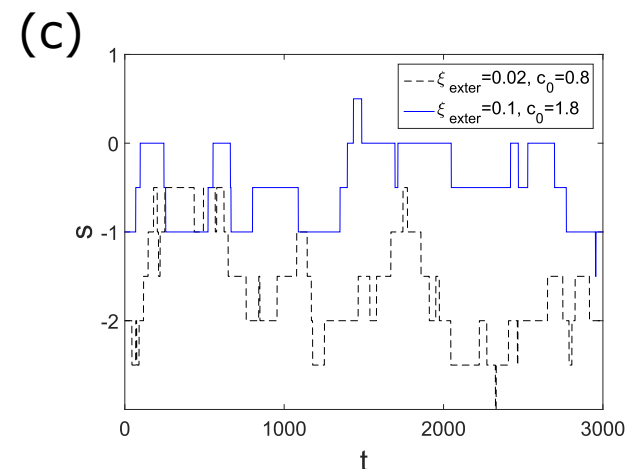
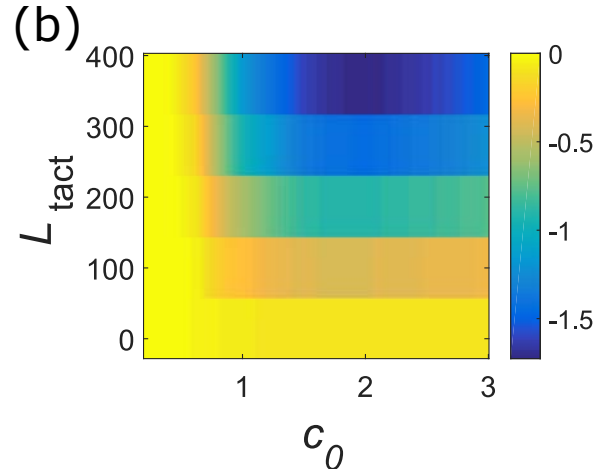
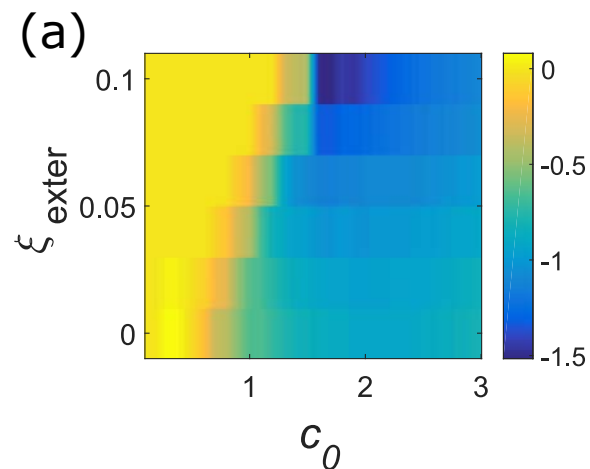
- $\theta_0$  – easy direction
- $w$  – strength of anchoring
- $\sigma$  – surface energy
- Here we introduce surface anchoring on the diffused interface via an external force  $F_{\text{exter}}$

# Numerical integration



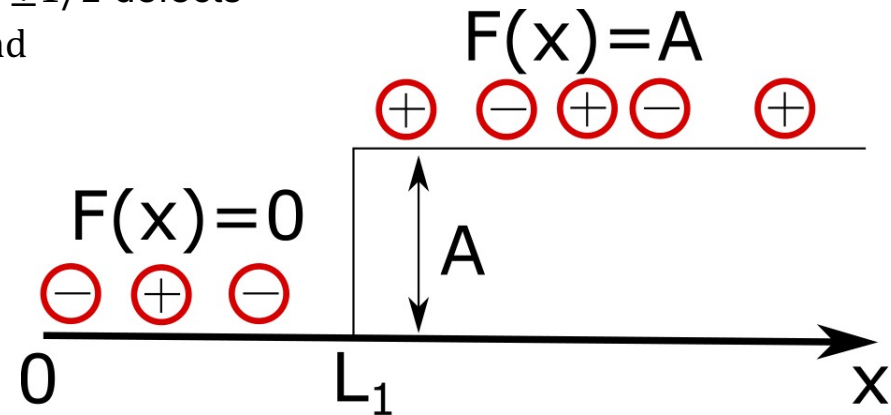
# Results

- Tactoids carry non-zero average topological charge
- Strong fluctuations
- Topological charge first increases, then decreases with the bacteria concentration
- Topological charge increases with tactoid's size



# Different defect mobility → charging

- Charging of tactoids due to different mobility of  $\pm 1/2$  defects
- Topological defects with the same sign attract and with different repel
- $+1/2$  defects have higher mobility
- Potential barrier at the I-N interface
- $P^+, P^-$  concentrations of  $+$  and  $-$  defects
- $P^+, P^-$  proportional to tact concentration  $c_0$



$$\partial_t P^+ = D^+ \partial_{xx} P^+ - \partial_x \left( \left( -F'(x) + \mu \int_0^\infty \frac{P^+(x', t) - P^-(x', t)}{x - x'} dx' \right) P^+ \right)$$

$$\partial_t P^- = D^- \partial_{xx} P^- - \partial_x \left( \left( -F'(x) + \mu \int_0^\infty \frac{P^-(x', t) - P^+(x', t)}{x - x'} dx' \right) P^- \right)$$

# Asymptotic solution

- Steady-state equation for  $P^-$ :

$$D^- \partial_{xx} P^- - \partial_x \left( \left( -F'(x) + \mu \int_0^\infty \frac{P^-(x') - P_0}{x - x'} dx' \right) P^- \right) = 0$$

- Solution for weak interaction:  $\mu = \varepsilon \mu_1$ ,  $P^- = P_0^- + \varepsilon P_1^- + \dots$

$$s = P_0 \left( 1 - e^{A/D^-} \right) \left( 1 - \frac{\mu}{D^-} P_0 \log(L/L_1) e^{A/D^-} \right)$$

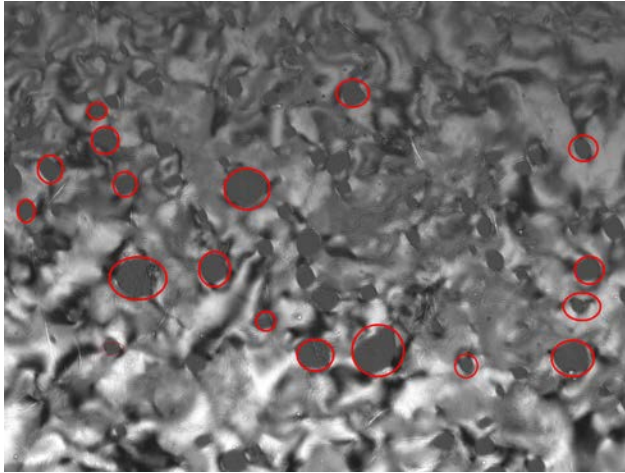
- Describes both isotropic and nematic tactoids

# Experimental Verification

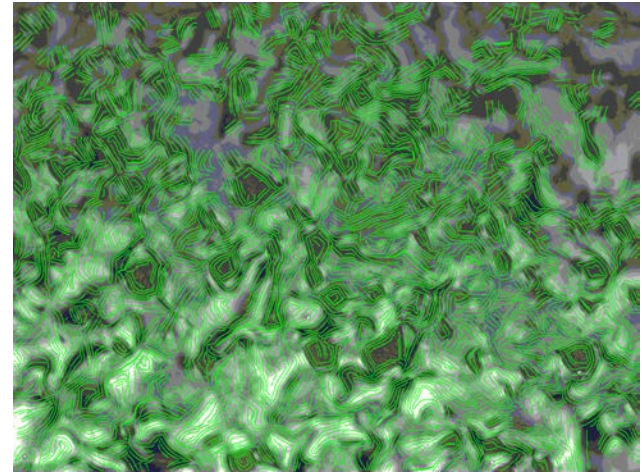
- LC is quenched in the biphasic domain by rapid heating
- Bacteria orientation is extracted from the microscopy images
- 17 tactoids are processed

# Comparison with the theory

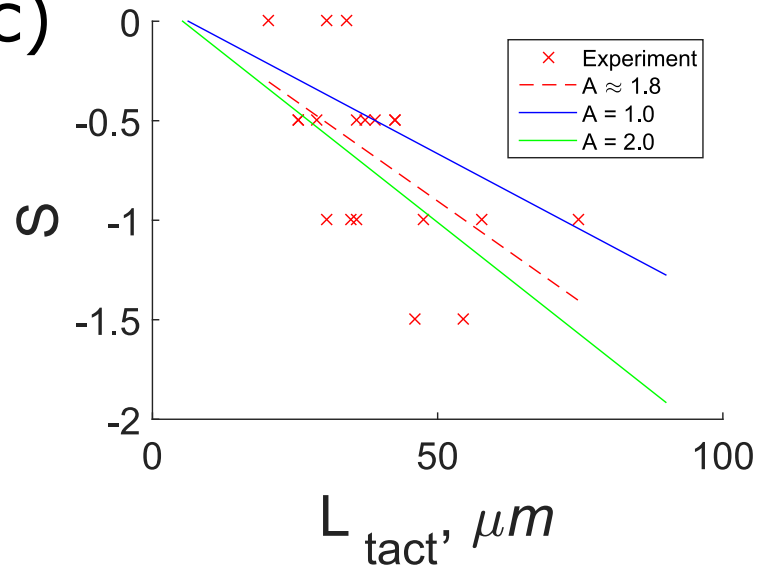
(a)



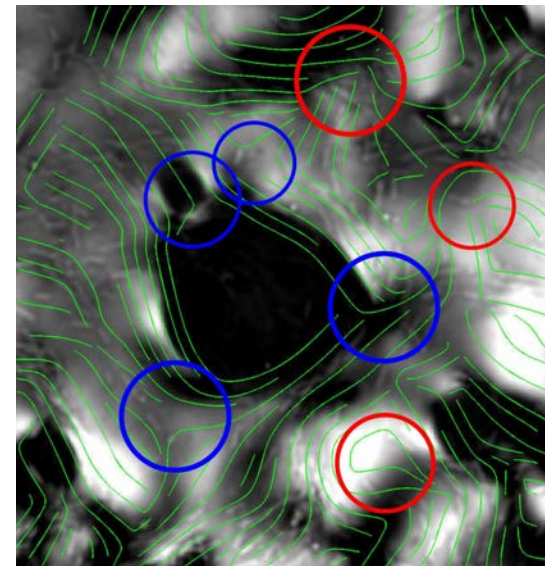
(b)



(c)

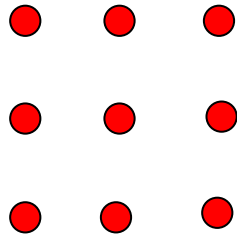


(d)



# Pinning of active topological defects

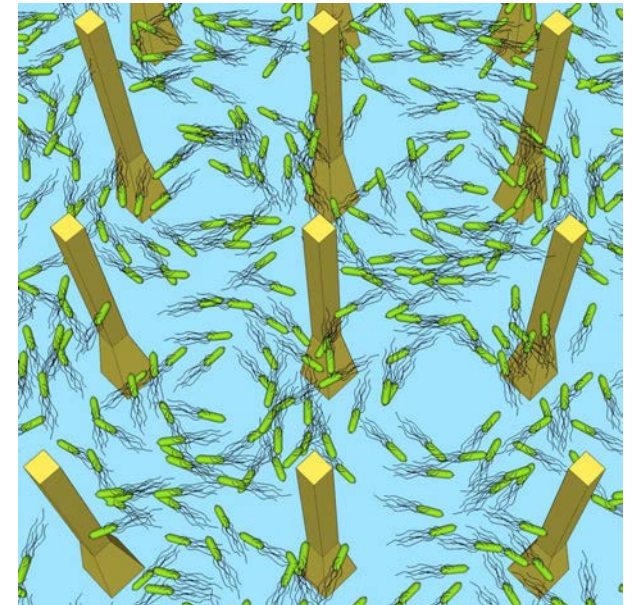
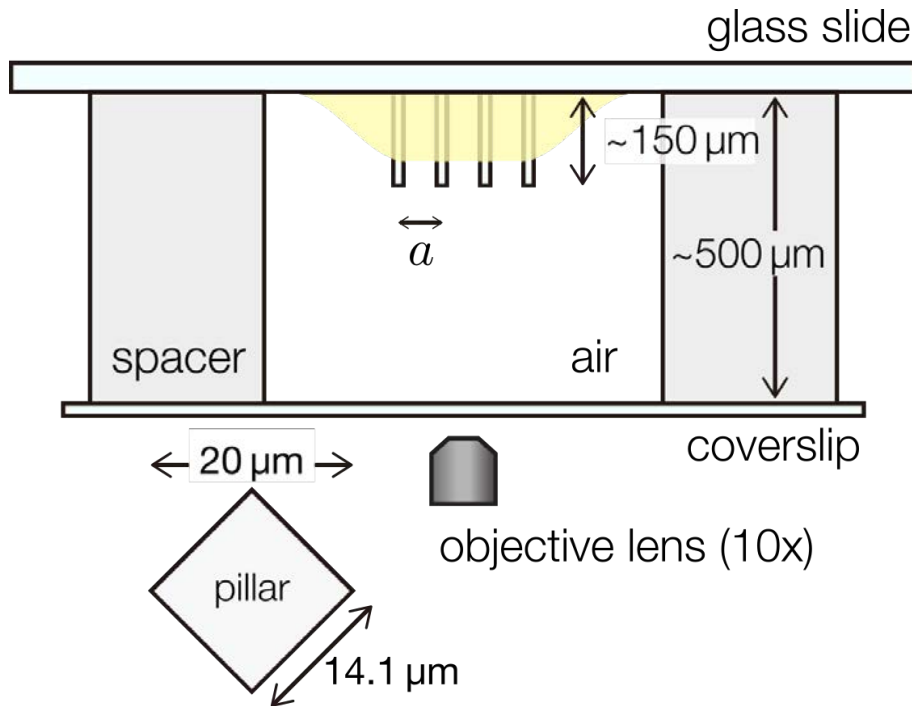
- Periodic array of artificial obstacles
- Only negative defects are pinned
- Positive defects freely move





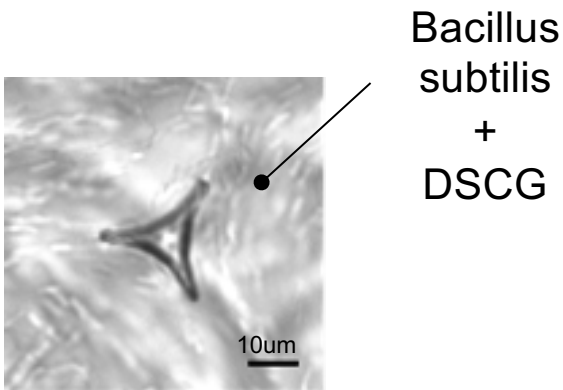
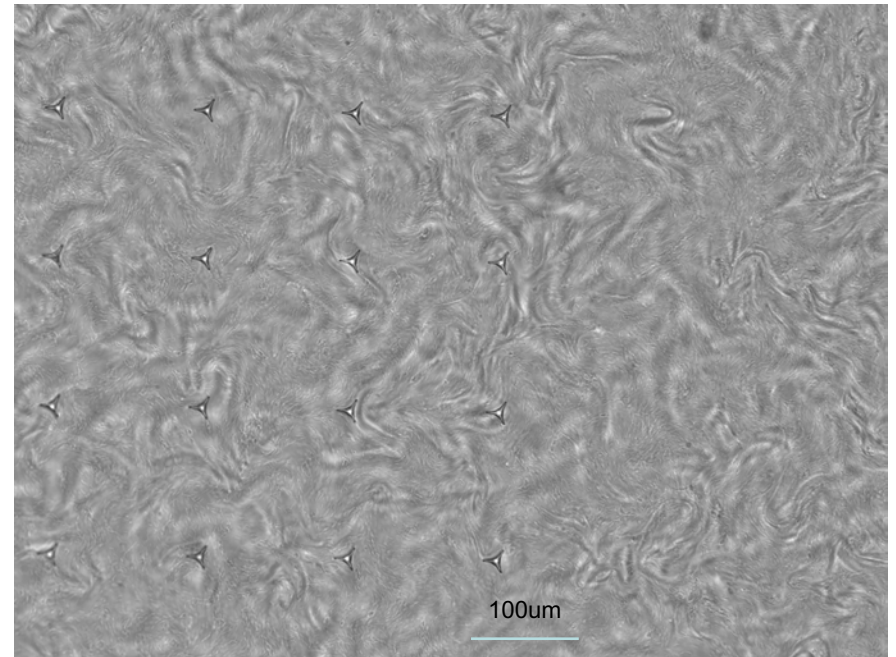
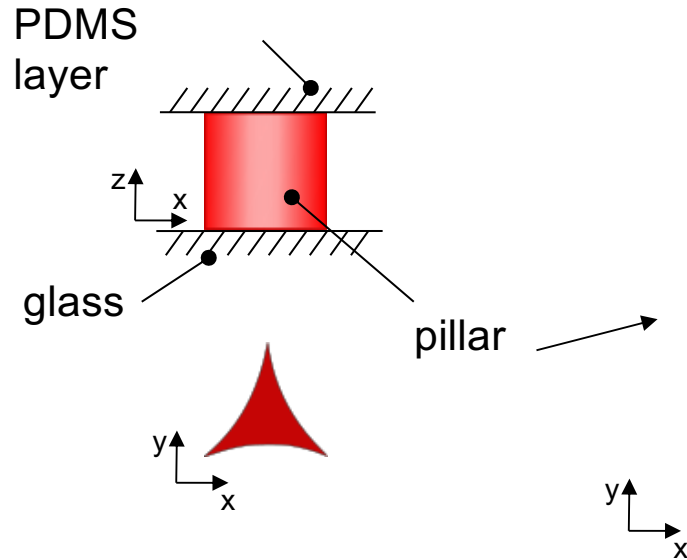
# Combing bacterial turbulence





- 3D printed pillar arrays with different spacing
- Two photon laser lithography system  
Nanoscribe, resolution 200 nm

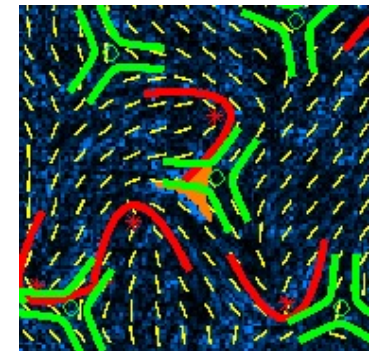


Nishiguchi et al IA, Nature Comm 2018

# Pillars in Liquid Crystals

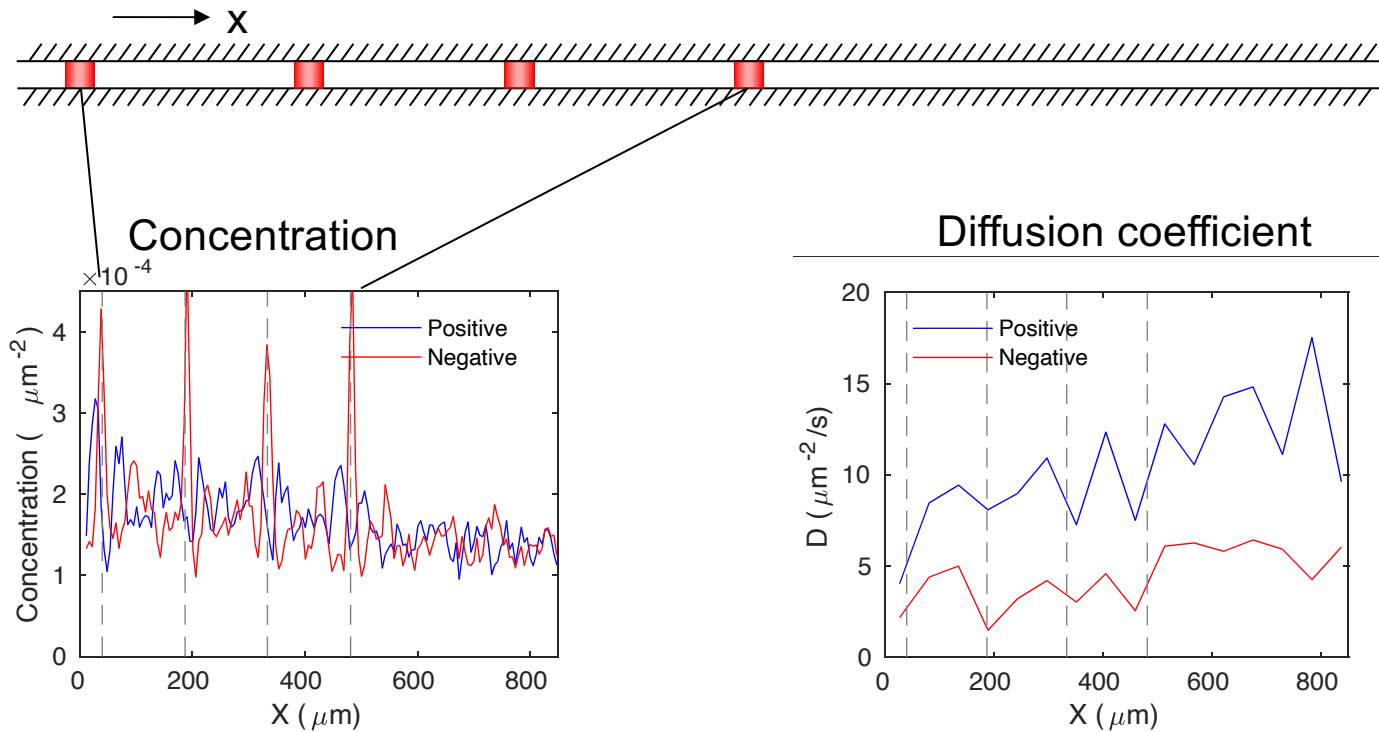


- Pillar 
- Nematic field lines 
- Nematic defects
  - +1/2 
  - 1/2 

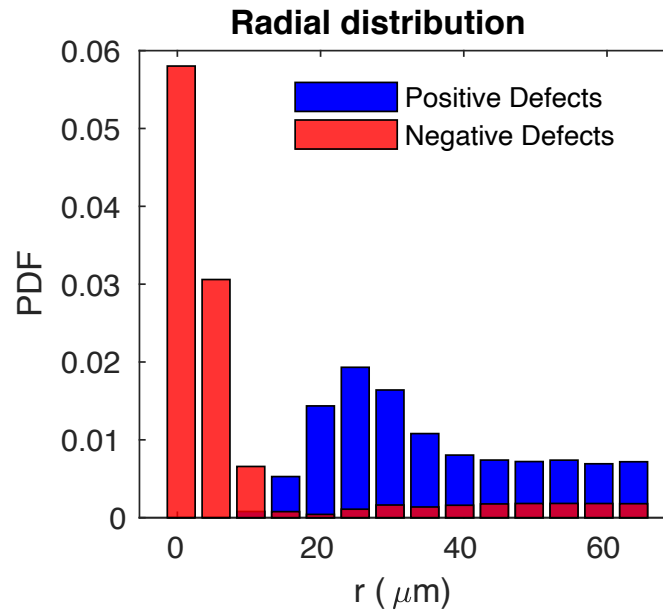
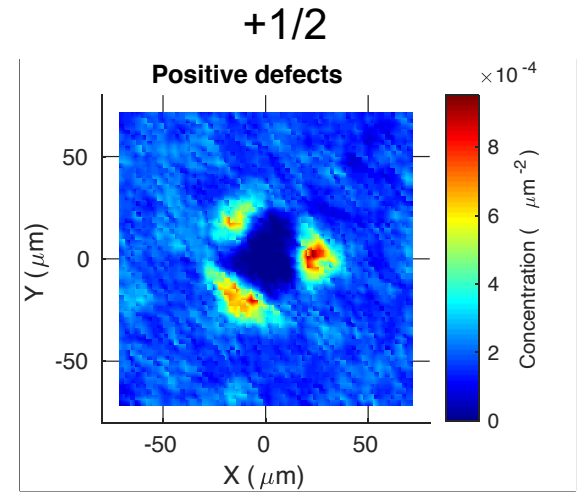
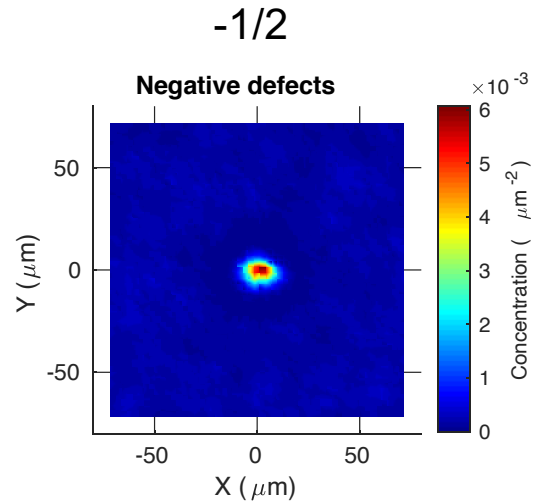
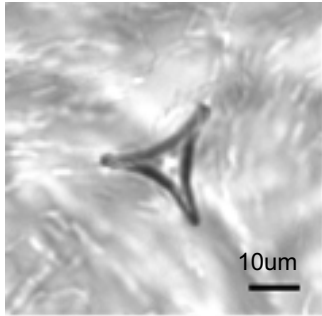


# With vs Without Pillars

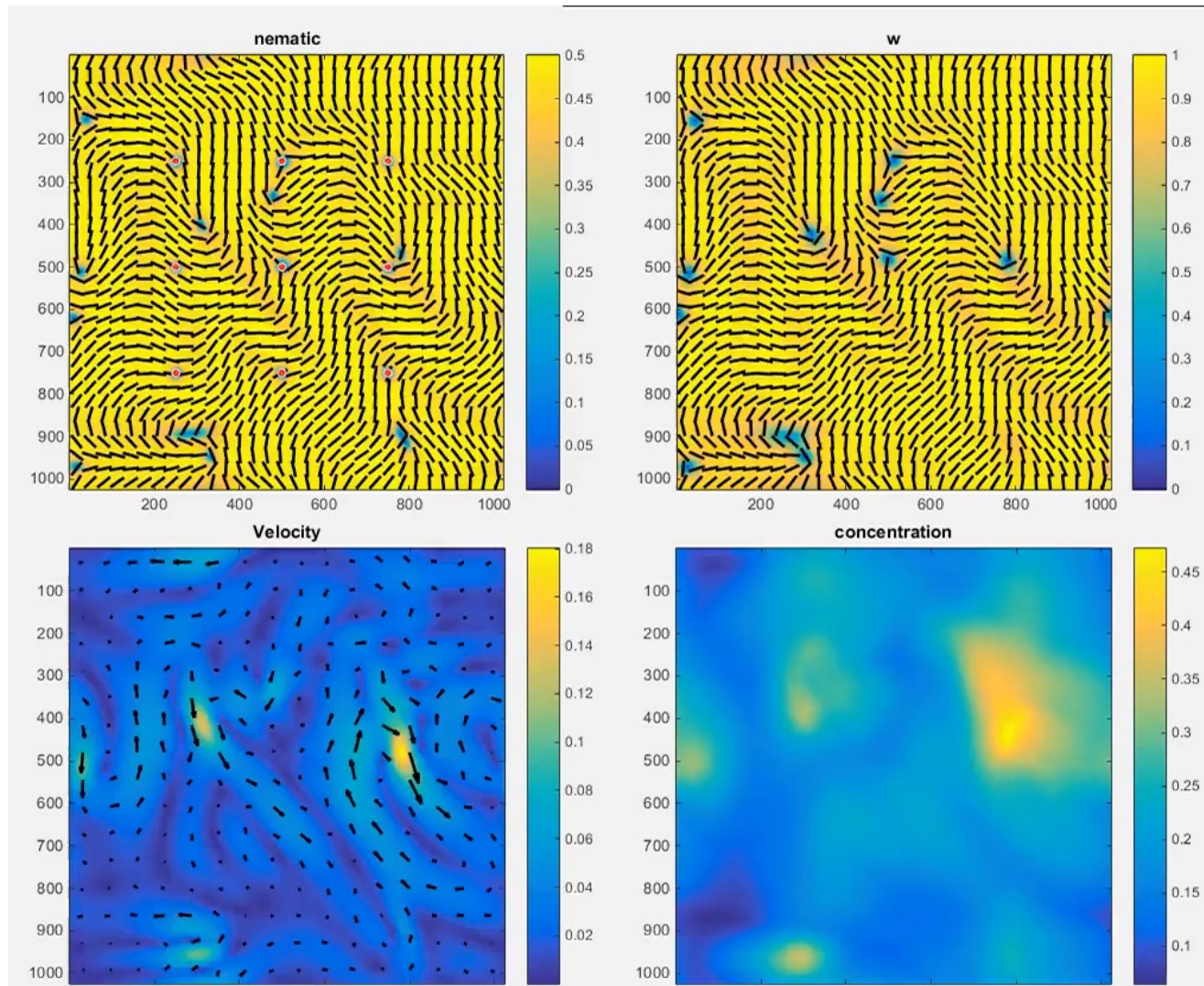
Average along Y



# Concentration around pillars

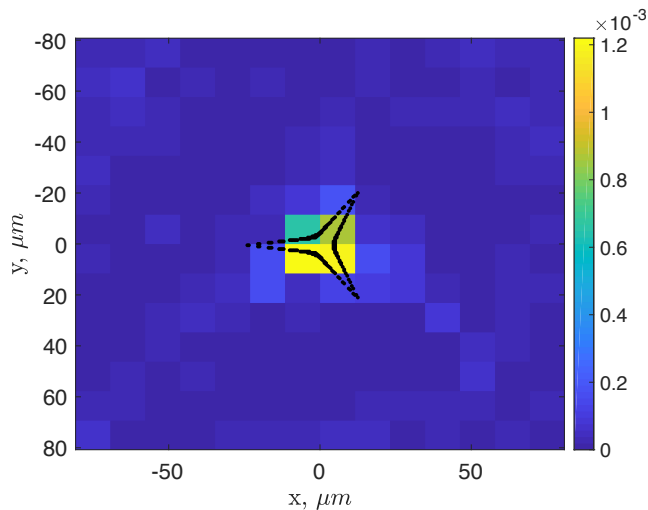


# Simulation results

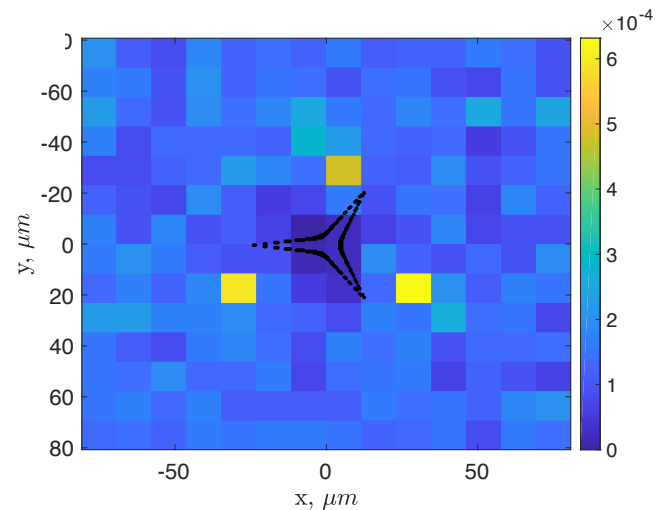


# Simulation results

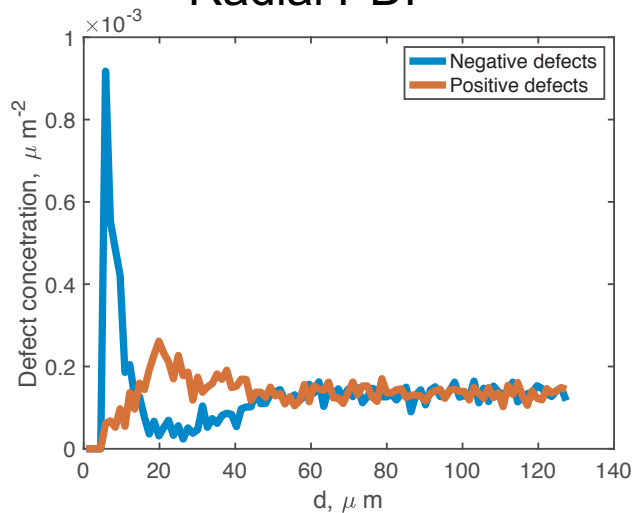
-1/2



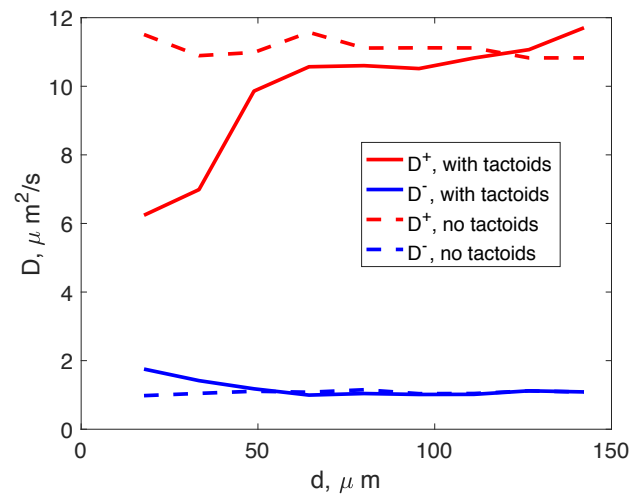
1/2



Radial PDF

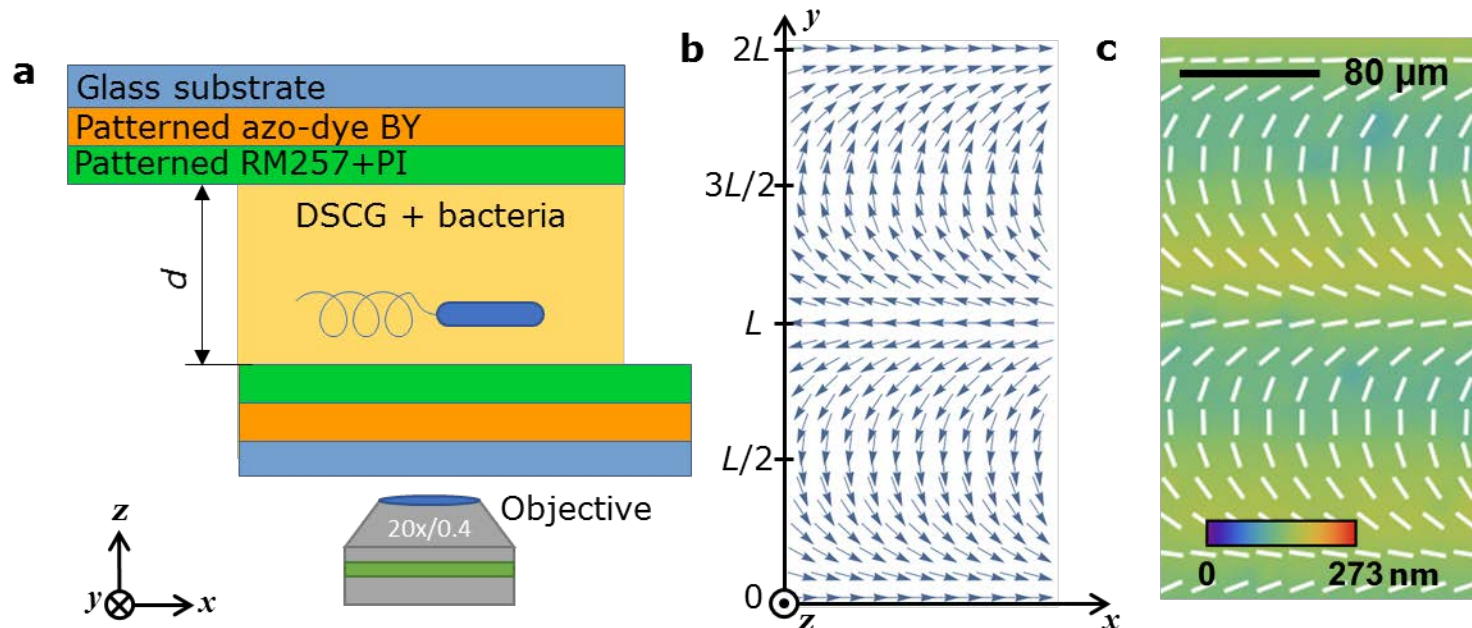


Diffusivity



# Formation of Polar Bacterial Jets

Photo-patterned nematic: Arrays of C-stripes



# Director pattern

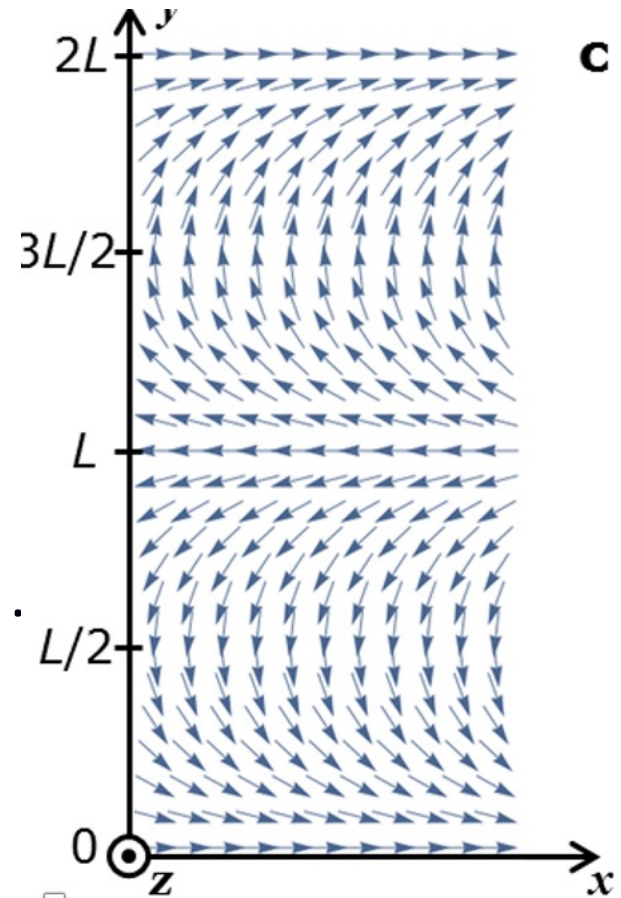
$$\hat{\mathbf{n}}_0 = \left[ \cos(\pi y / L), -\sin(\pi y / L), 0 \right]$$

Pure splay deformations are located at

$$y = 0, \pm L, \pm 2L, \dots$$

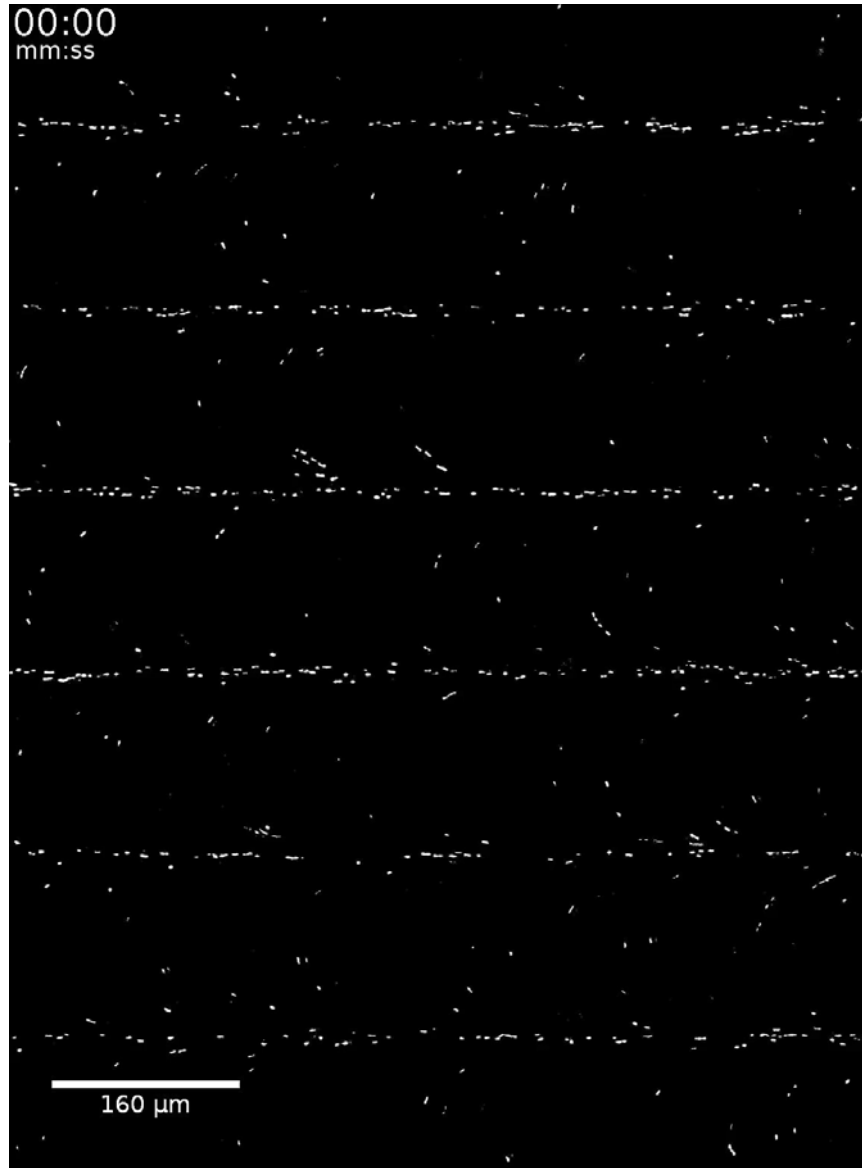
Bend Deformations

$$y = \pm L / 2, \pm 3L / 2, \dots$$





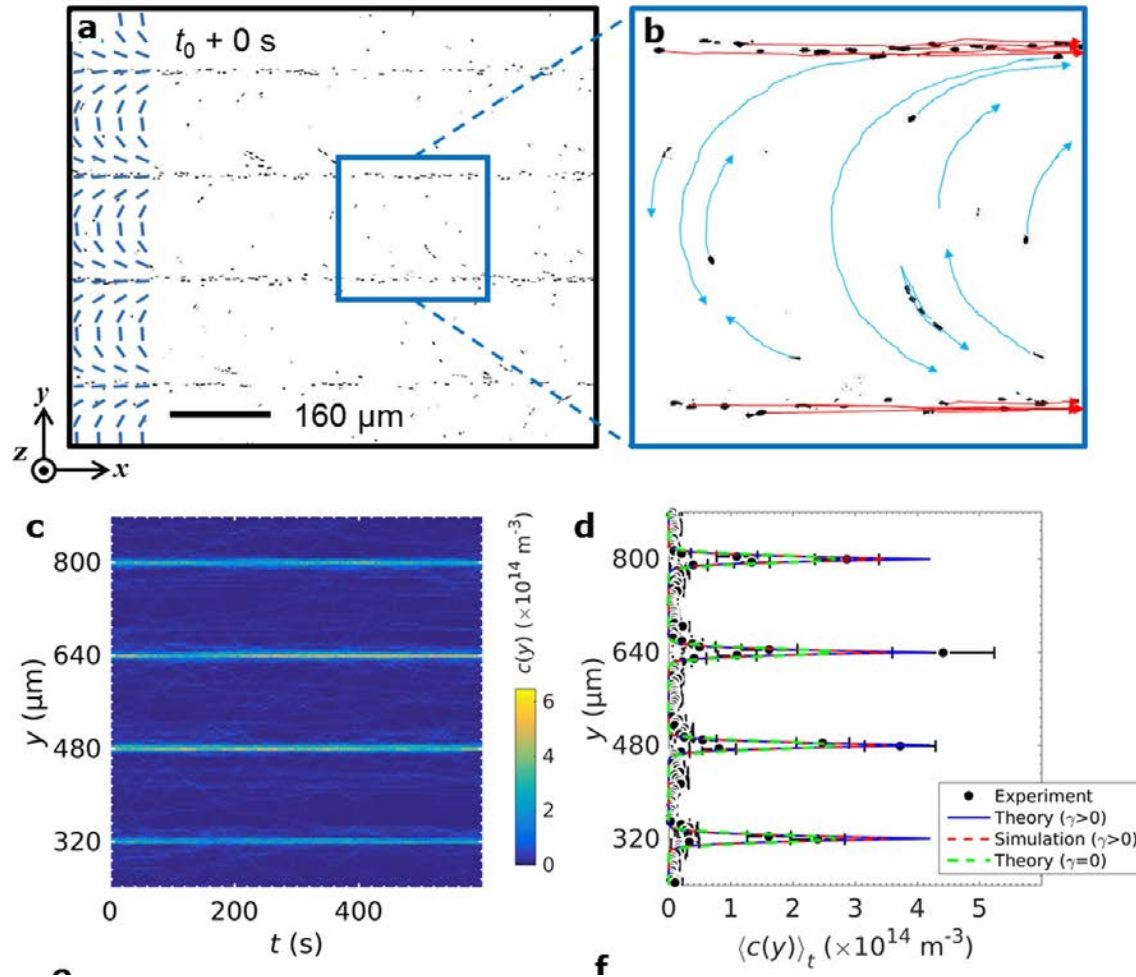
# Raw images of bacteria



# Undulations for higher concentration



# Bacteria Condense in Polar Jets



# Theoretical understanding: two bacterial populations

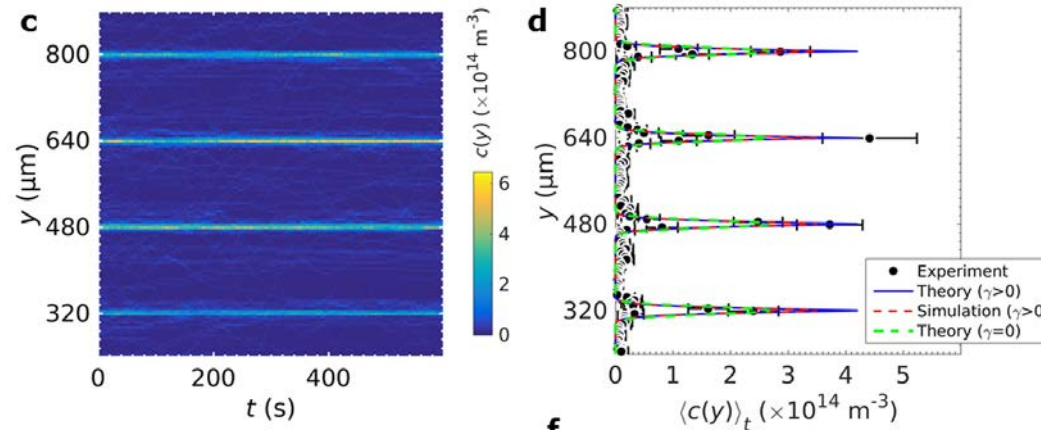
$$\partial_t c^\pm + \nabla \cdot (\pm v_0 \hat{\mathbf{n}} c^\pm + \mathbf{v}_f c^\pm) = \mp \frac{c^+ - c^-}{\tau} + D_c \nabla^2 c^\pm$$

- Infinite reversal time  $\tau$
- Neglect fluid flow
- Stationary solution
- Very dilute suspension – no realigning effect

# Suppression one of the population

- Analytical result

$$c^{\pm} = C^{\pm} \exp\left(\pm \frac{v_0 L}{\pi D_c} \cos \frac{\pi y}{L}\right)$$



- $c^+$  population is exponentially amplified
- $c^-$  population is exponentially suppressed
- However, the peaks are not sharp

# Less dilute suspension: realignment of director by bacteria

$$\partial_t \hat{\mathbf{n}} = -\hat{\mathbf{n}} \times \hat{\mathbf{n}} \times (\alpha \hat{\mathbf{n}}_0 + \gamma \mathbf{F})$$

Landau-Lifshitz equation ensures  $\hat{\mathbf{n}}^2 = 1$

$\hat{\mathbf{n}}_0$  prescribed director

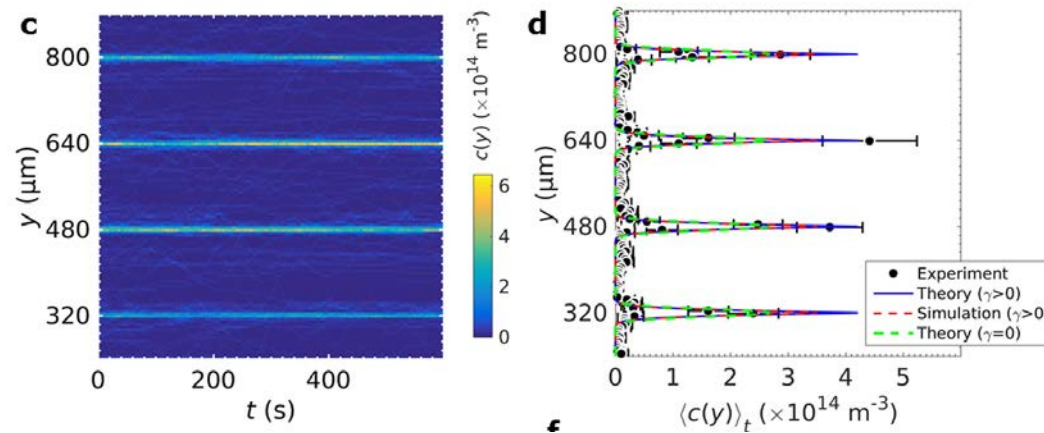
$\mathbf{F} = \nabla \cdot \boldsymbol{\sigma}$  active force created by bacteria

$\boldsymbol{\sigma} \sim c \mathbf{Q}$  – active stress,  $\mathbf{Q}$  – nematic OP

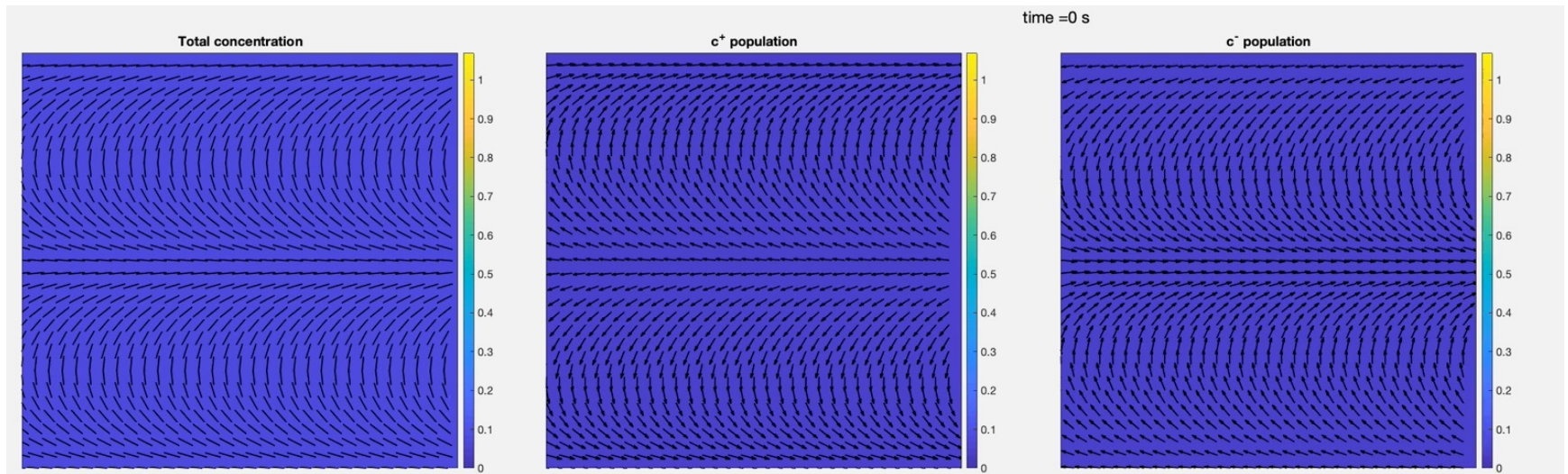
# Solution with sharp peaks

$$c^+ = - \frac{W \left\{ -\frac{\gamma v_0 U_0}{a D_c} C^+ \exp \left[ \frac{v_0 L}{\pi D_c} \cos(ky) \right] \cos(2ky) \right\}}{\frac{\gamma v_0 U_0}{a D_c} \cos(2ky)}$$

$W$  – Lambert W function

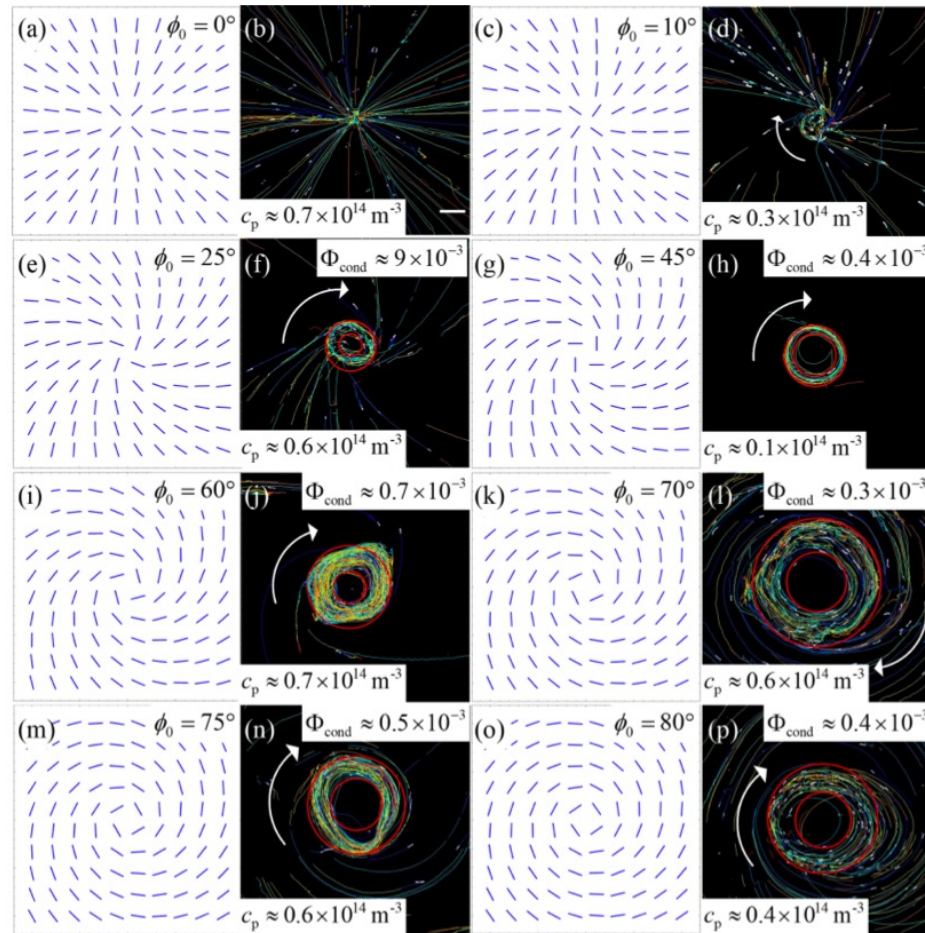


# Results of comp modeling





# Control of bacteria by spiral nematic vortices



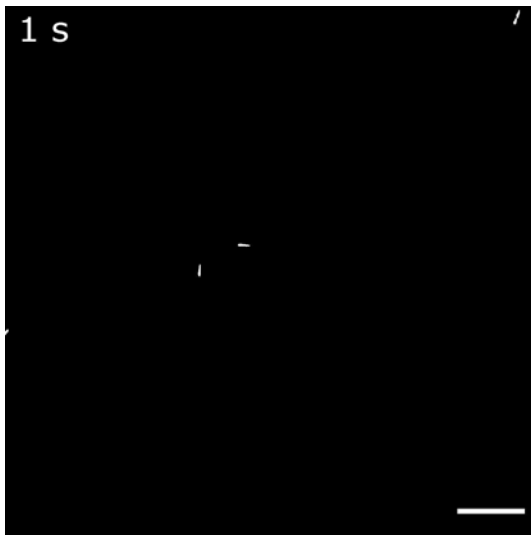
# Vortices and limit cycles

- Prepatterned director: spiral with different pitch
- Low concentration of bacteria – scattering from the center of the spiral
- Higher concentration: limit cycles
- Most stable cycles for the tilt angle  $45^\circ$

# Stable cycles

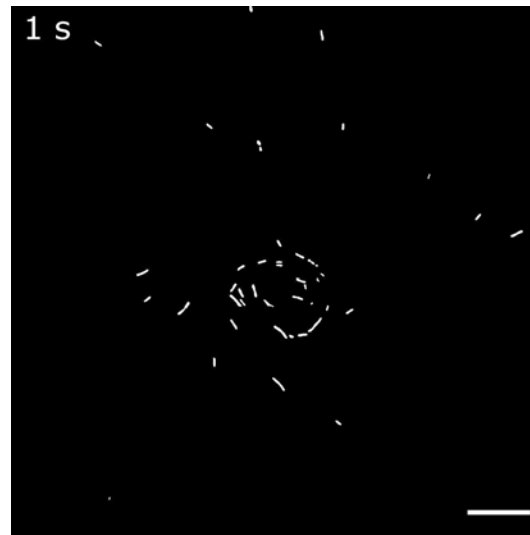
Low concentration

$$\phi = 25^\circ$$

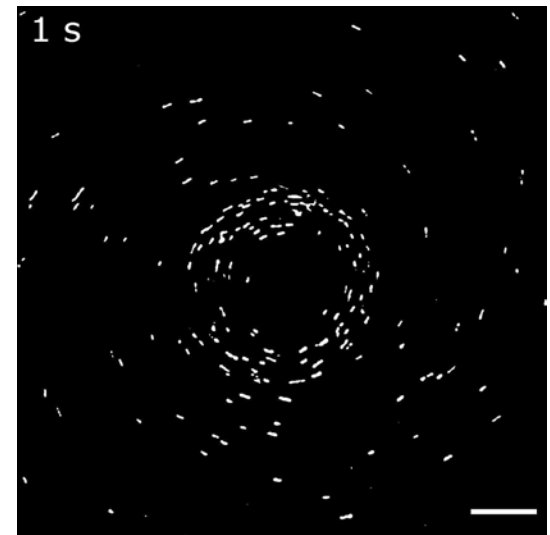


High concentration

$$\phi = 25^\circ$$



$$\phi = 75^\circ$$



# Agent-based simulations

- Bacteria swim with the speed  $V$  along the director

$$\partial_t \theta = \gamma_B \sin[2(\phi_0 - \theta)]$$

- Bacteria realign the director

$$\partial_t \hat{\mathbf{n}} = -\hat{\mathbf{n}} \times \hat{\mathbf{n}} \times (\gamma_s \hat{\mathbf{n}}_0 - 2a\mathbf{F}_{\text{act}} + K\Delta\hat{\mathbf{n}})$$

- Active Force  $\mathbf{F}_{\text{act}} = \nabla \cdot \sum \boldsymbol{\sigma}_{\text{act}}$

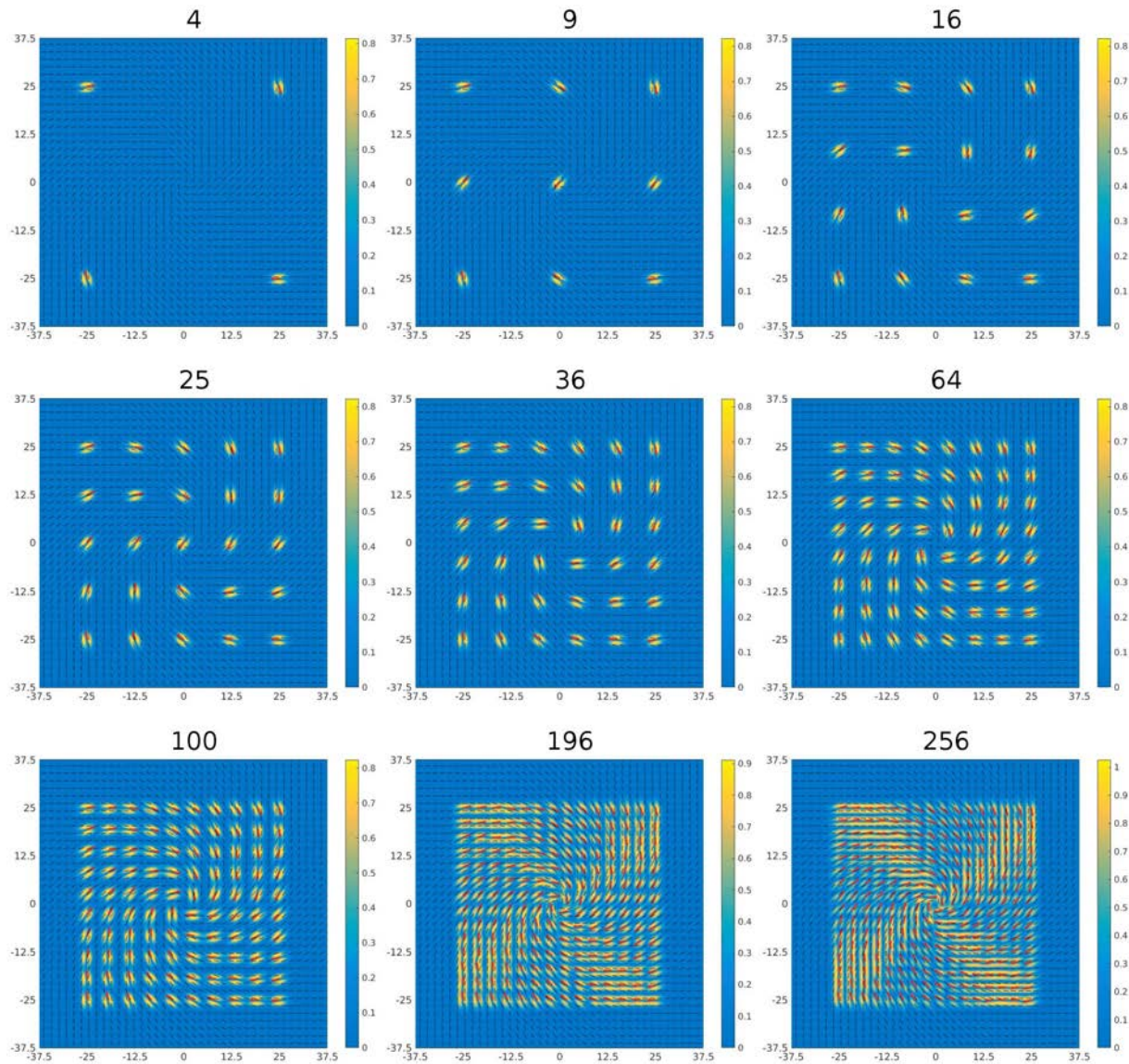
- Active stress  $\boldsymbol{\sigma}_{\text{act}} = -\lambda \mathbf{Q}_B f(\mathbf{r})$

- Nematic tensor  $\mathbf{Q}_B = \hat{\mathbf{p}}\hat{\mathbf{p}} - \mathbf{I}/2$

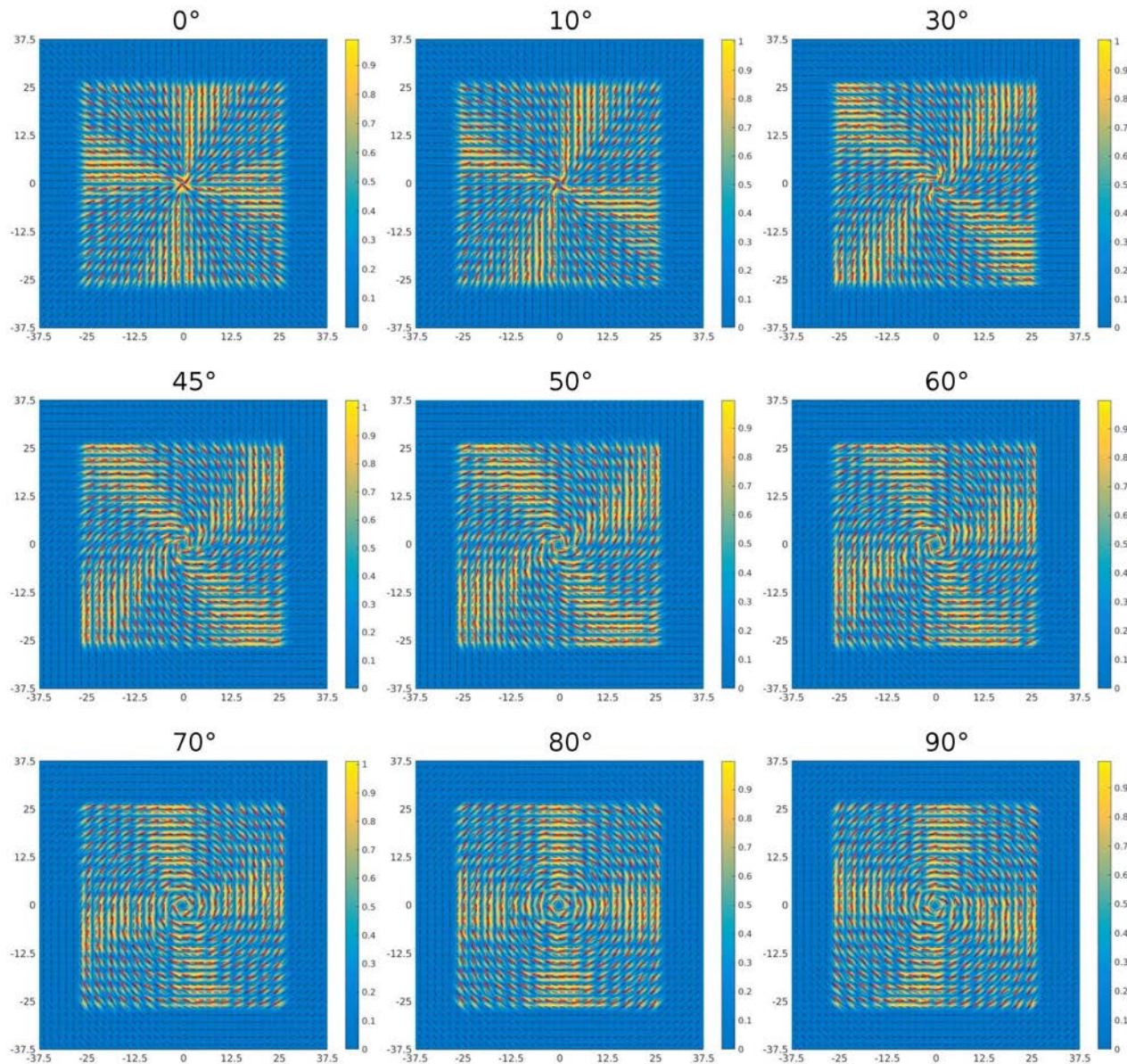
- $K$ - elastic constant

- $f$ - shape of the bacterium

# Function of the Number of Bacteria



# Function of the Tilt Angle



# References

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- Genkin, Sokolov, Lavrentovich, IA PRX 2017
- Genkin, Sokolov, IA, NJP , 2018
- Taras Turiv, Runa Koizumi, Kristian Thijssen, Mikhail M. Genkin, Hao Yu, Chenhui Peng, Qi-Huo Wei, Julia M. Yeomans, IA, Amin Doostmohammadi, Oleg D. Lavrentovich, Nature Physics 2020
- Runa Koizumi, Taras Turiv, Mikhail M. Genkin, Robert J. Lastowski, Hao Yu, Irakli Chaganava, Qi-Huo Wei, IA, Oleg D. Lavrentovich, PR Research 2020

Review paper to appear: Bacterial Active Matter,  
Reports on Progress in Physics, 2022

# Summary:

- LLC exhibit coupling between activity and topology
- Computational model is in faithful agreement with experiment
- Experiments fully support our theoretical predictions
- Bacteria accumulate in the cores of  $\frac{1}{2}$  defects and expelled from the cores of  $-\frac{1}{2}$  defects
- Asymmetric pinning
- Control of bacterial motion by surface patterning