Active Matter : from liquids to solids

from Collective Motion
to Collective Actuation

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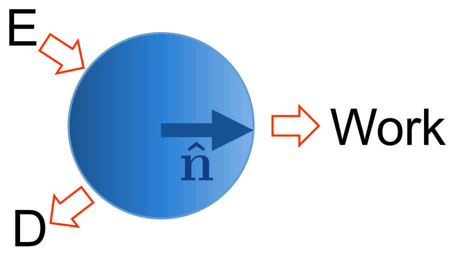
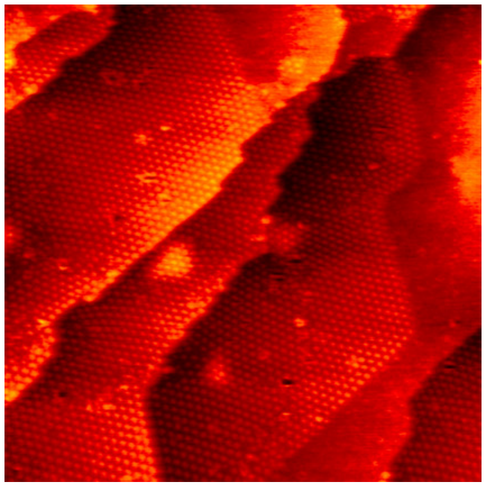
What is active matter ?

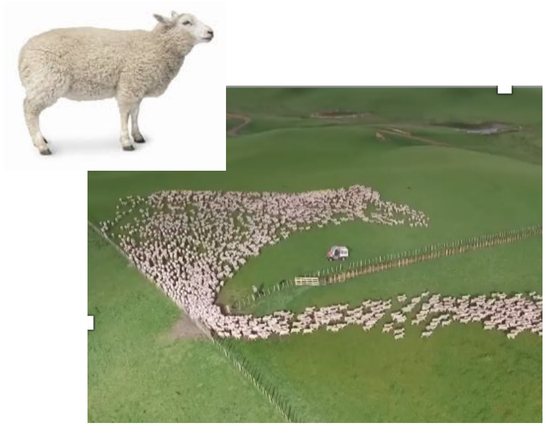
- The matter of which atoms are active units
- Each active unit follows dynamics with
  - broken time reversal symmetry
  - broken space isotropy

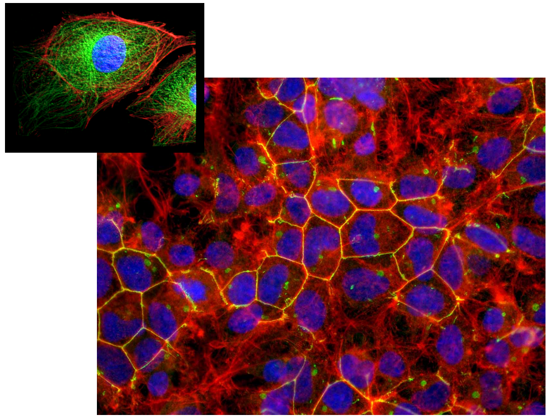




Why is Active Matter interesting for physicists?

- The simplest out of equilibrium matter phases, with new physics



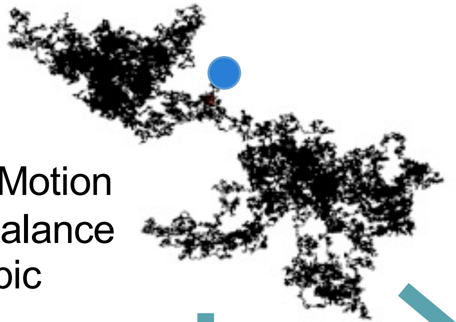


- In active fluids
  - mechanical pressure is not a state variable
  - liquid-gas phase separation takes place in purely repulsive systems
  - macroscopic flows emerge in the absence of external gradient : collective motion

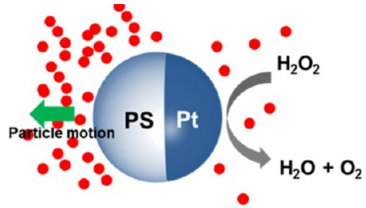
- In active solids
  - spontaneous flows can take place in crystalline structure
  - selective & collective oscillations emerge in overdamped linear elastic systems

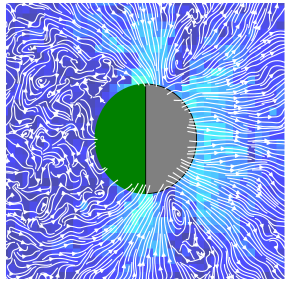
- It offers a unique point of view on traditional matter

Active matter outside of the realm of the living world or robotics

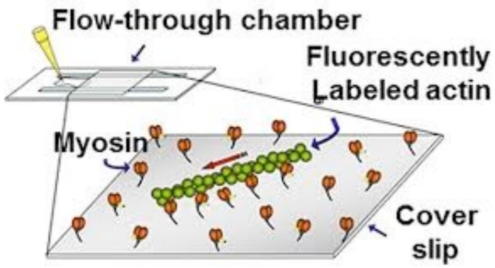


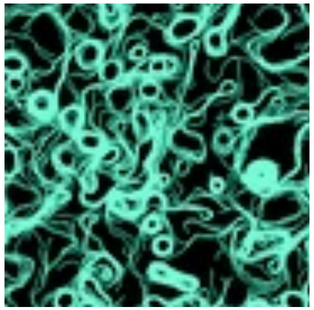
Brownian Motion
Detailed Balance
Isotropic





Phoretic Motion
Janus Colloid





Motility Assay

Shaken Beads
No Detailed Balance
But Isotropic

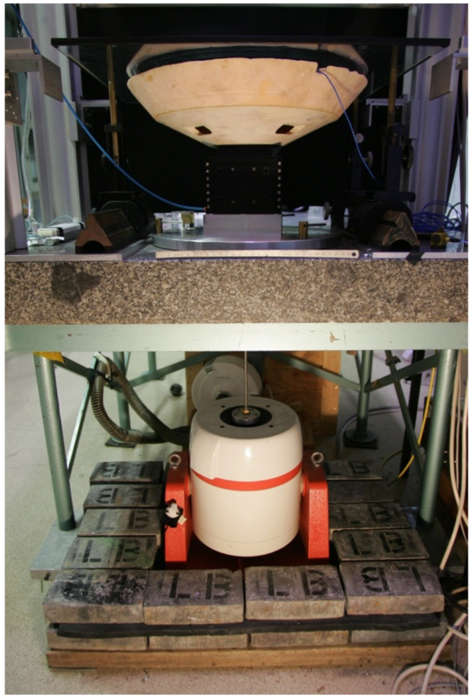
nematic

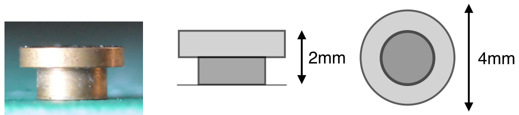
Shaken rice grains

polar

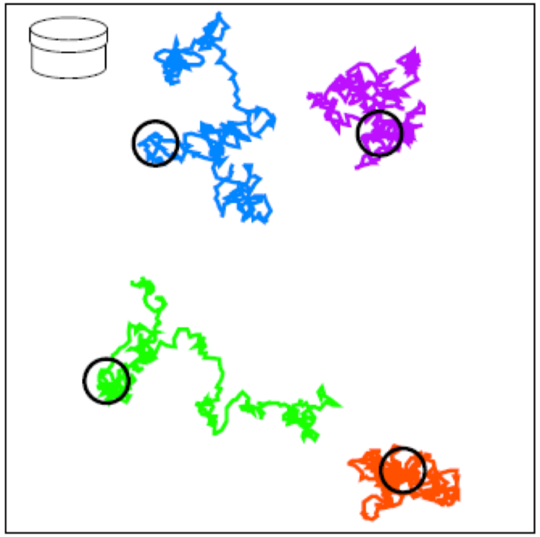
Shaken sunflower seed

The walking grains : from diffusion to self propulsion

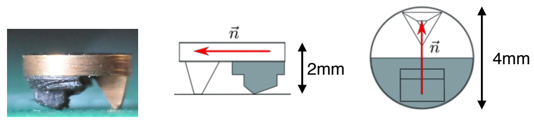




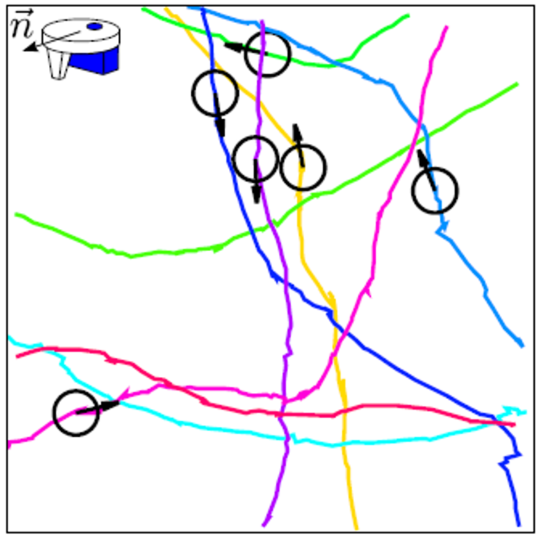
Isotropic particles



Brownian like motion



Polar particles



Directed Random Walk

Outline: from active liquids to active solids

- Active fluids : a brief overview with a focus on collective motion
  - mechanical pressure is not a state variable
  - liquid-gas phase separation takes place in purely repulsive systems
  - collective motion

- Active solids :
  - spontaneous flows can take place in crystalline structure
  - selective & collective actuation emerges in linear elastic systems

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Pressure in passive and active system

- At equilibrium
  - Mechanical force against a wall $P_{mech} = \dfrac{F_{wall}}{S}$
  - Hydrodynamics : Flux of momentum $\partial_t g + div(\sigma) = f_{ext} \qquad P_{hy} = tr(\sigma)/d$
  - Thermodynamics $P_{th} = -\dfrac{\partial \mathcal{F}}{\partial V}$

  - Momentum conservation => $P_{hy} = P_{mech}$
  - Boltzmann distribution => $P_{th} = P_{mech}$
  - In the thermodynamic limit: EOS $P_{hydro} = P_{mech} = P_{th} = f(\rho, T)$

- Active systems
  - No Momentum conservation => $P_{hy} \overset{?}{=} P_{mech}$
  - No Boltzmann distribution => $\cancel{P_{th}} = P_{mech}$

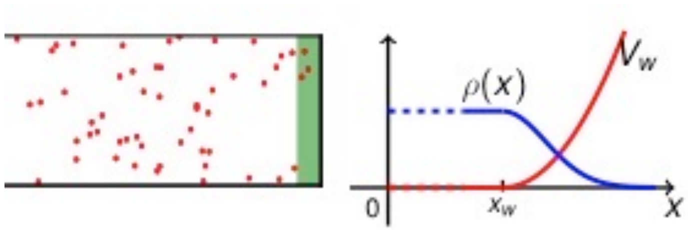
Mechanical Pressure in active systems : theory

- Following the Virial theorem introducing an active part

$$-\left\langle \sum_i f_i^{ext} r_i \right\rangle = E_{kin} + \left\langle \sum_i f_i^{int} r_i \right\rangle + \left\langle \sum_i f_i^{act} r_i \right\rangle$$

NB: "swim pressure" depends on interaction because f_i^{act} aims at $|v| \to v_0$

- Pressure against a wall



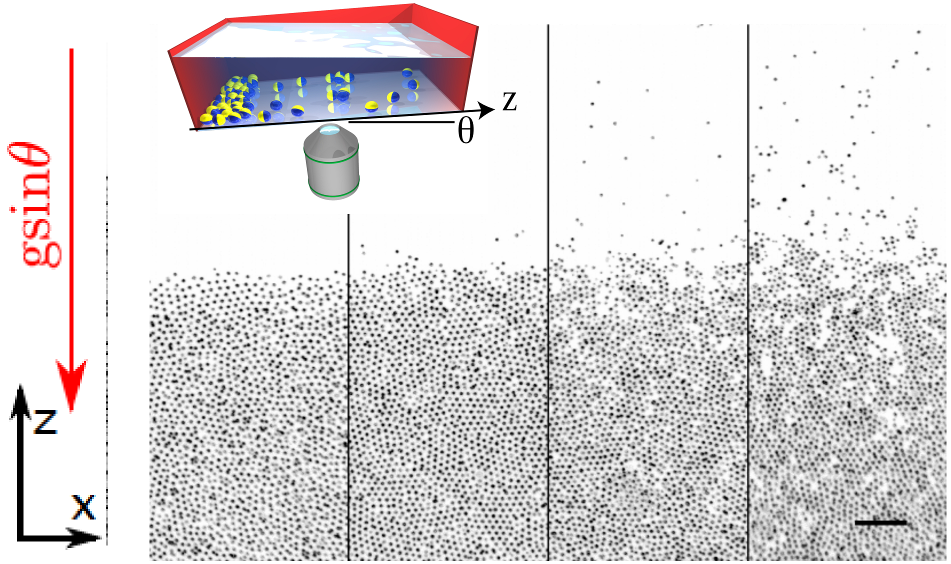
$$P = \int dx\, \rho(x) V_w'(x)$$

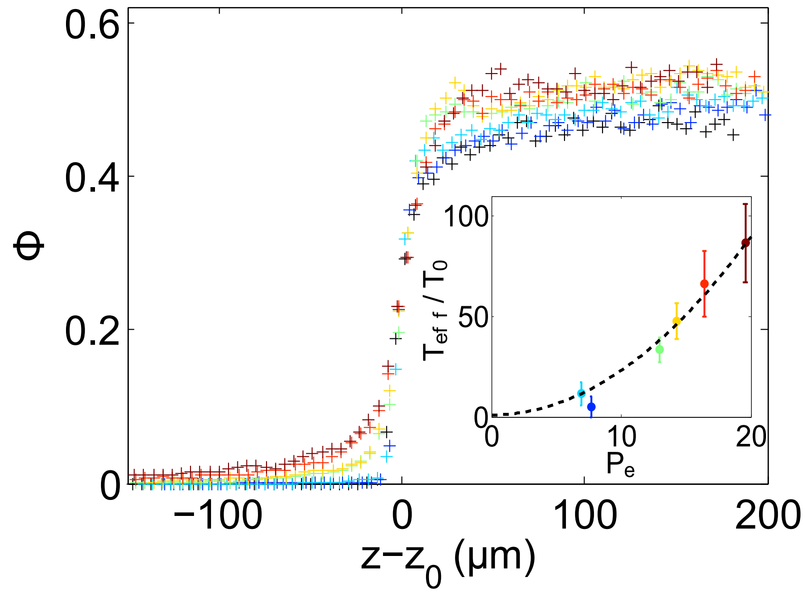
- In both cases
  - ABP without interaction $p_s^0 = \rho \dfrac{v_0^2}{2\mu D_r}$; with interaction $p_s = \rho \dfrac{v_0 v(\rho)}{2 D_r \mu}$
  - Other than ABP : no EOS

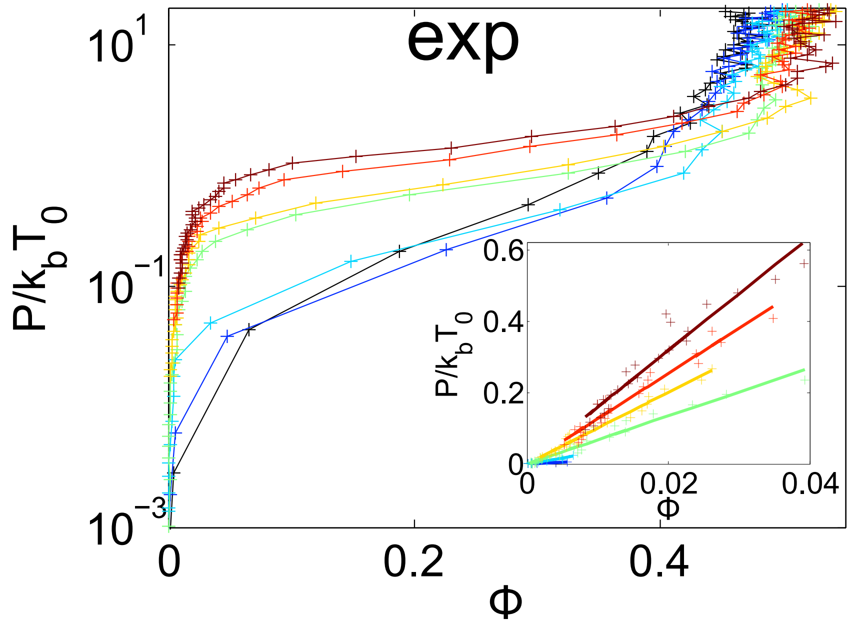
Yang, Manning & Marchetti, Soft Matter **10**, 6477 (2014)
Mallory, Sarić, Valeriani & Cacciuto, PRE **89**, 052303 (2014)
Takatori & Brady, PRL **113**, 028103 (2014)
Solon, Fily et al Nature Phys. **10.1038** (2015)
Solon *et al.* PRL (2015)
Nicolas, Solon, et al. PRL **117**, 098001 (2016)
Winkler et al. Soft Matter, **11**, 6680 (2015)

Hydrostatic Pressure in active system : experiments

Ginot et al. Phys Rev X 5, 011004 (2015)



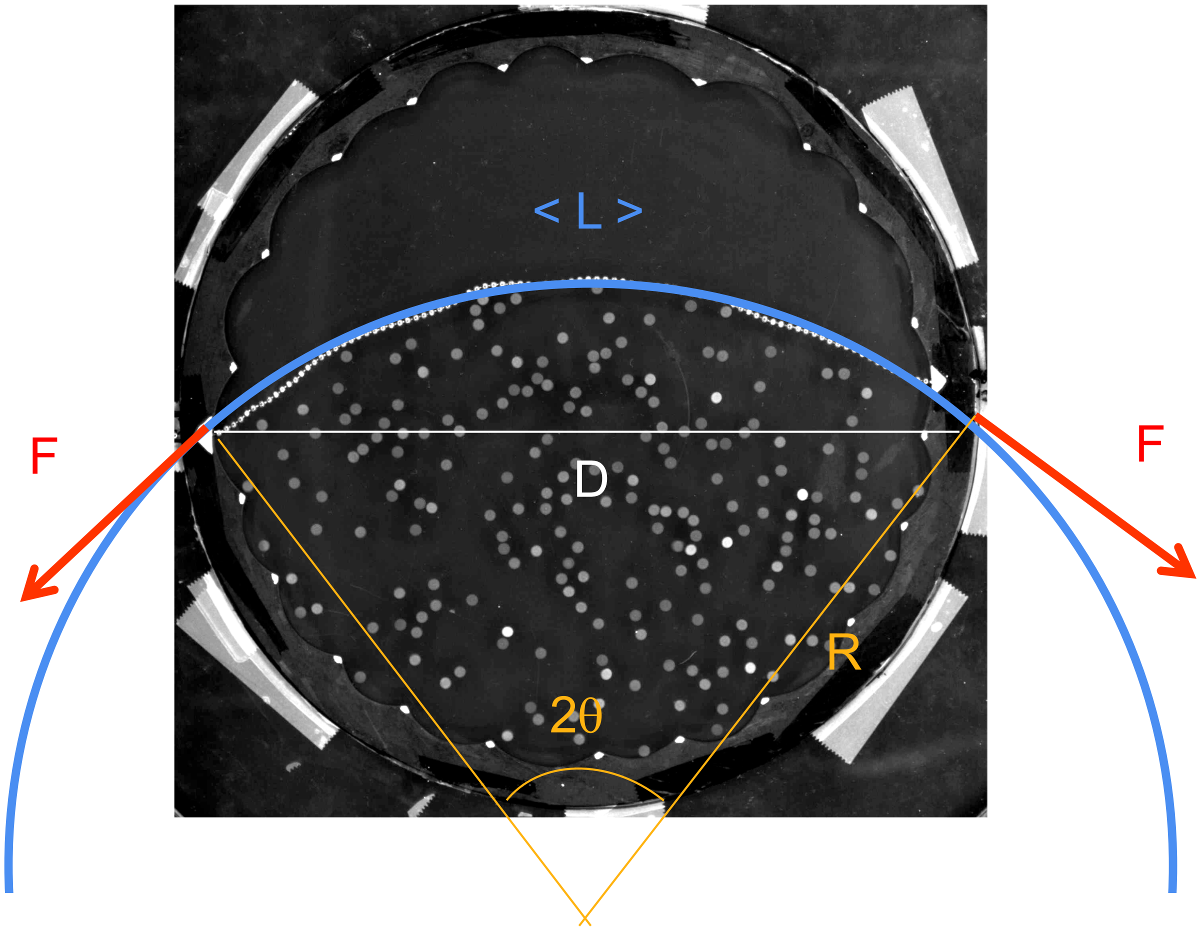




$$\Pi(z) = \frac{mg\sin\theta}{\pi R^2}\int_z^L dz'\phi(z')$$

$$P(z) = \Pi(z) - \Pi_0$$

Measuring pressure in the vibrated grains : the barometer



Torque free membrane

Mechanical Equilibrium

$$P \times D = 2F sin(\theta)$$

Geometry

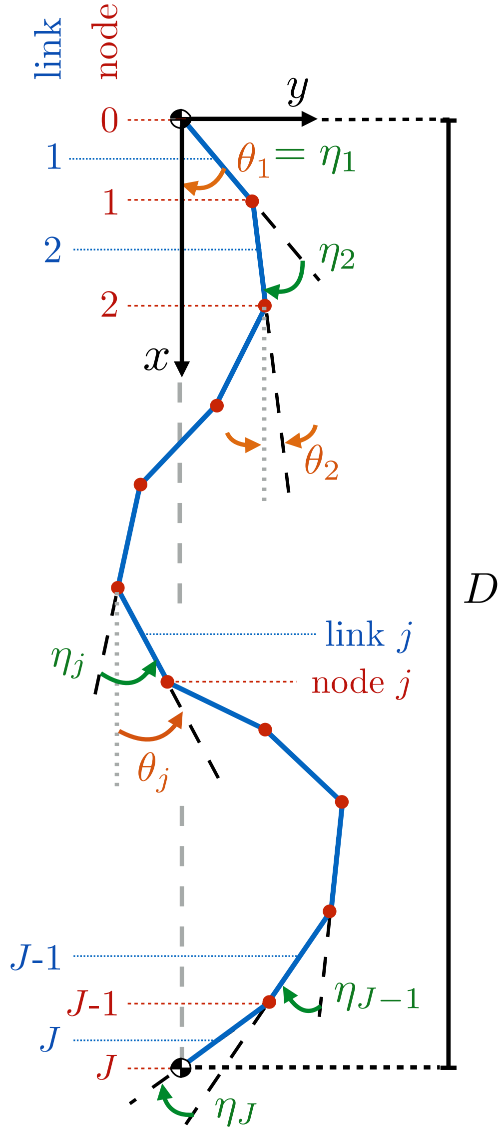
$$\frac{D}{2R} = sin(\theta)$$

Laplace Law

$$P = F/R$$

Need for $F(\langle L \rangle)$ the mechanical law of the membrane

A model entropic membrane : the necklace

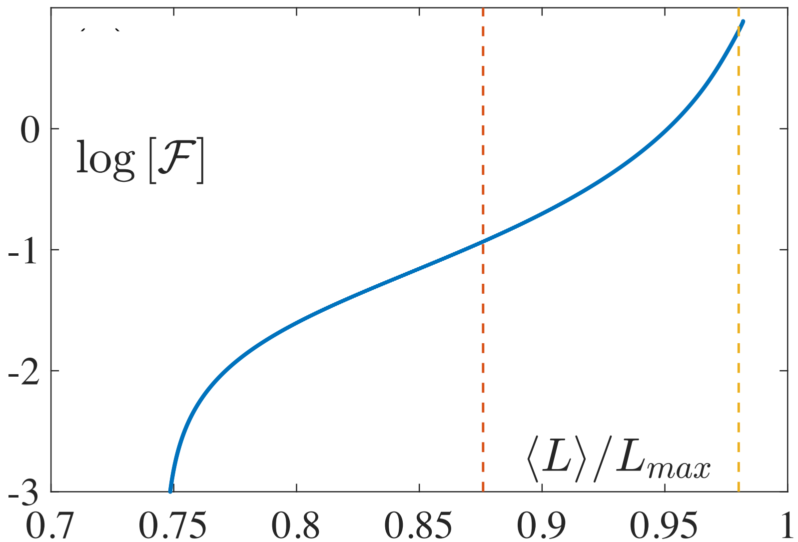


- ✓ J = 92 or 147 points
- ✓ Rigid rods $L_{max} = J\ell$
- ✓ Torque free ball joints
- ✓ Maximal opening $\eta_{max} = \pi/8$

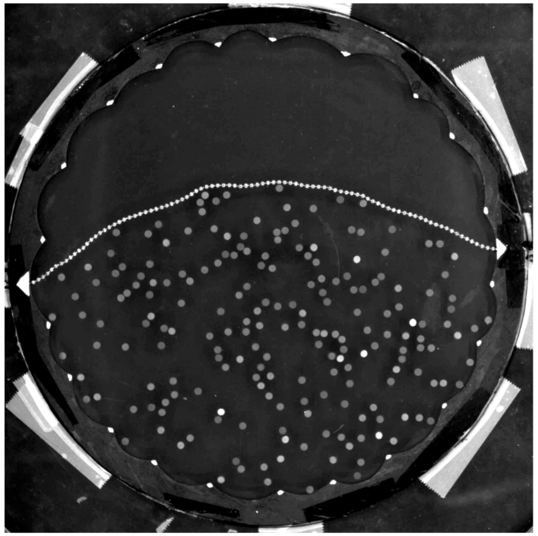
Monte Carlo sampling of Z

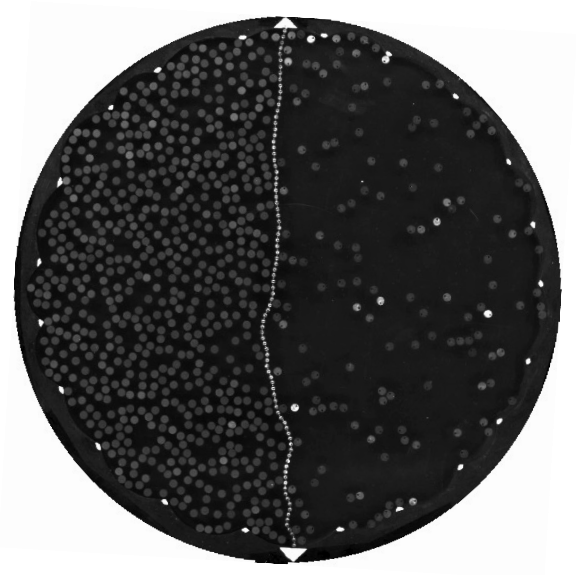
$$\langle L\rangle(F, J) = \frac{1}{\beta}\left[\frac{\partial}{\partial F}\ln(Z)\right]_J$$

$$\beta\ell F = \mathcal{F}\left(\frac{\langle L\rangle}{L_{max}}\right)$$



Mechanical pressure for Isotropic vs. Polar Disks

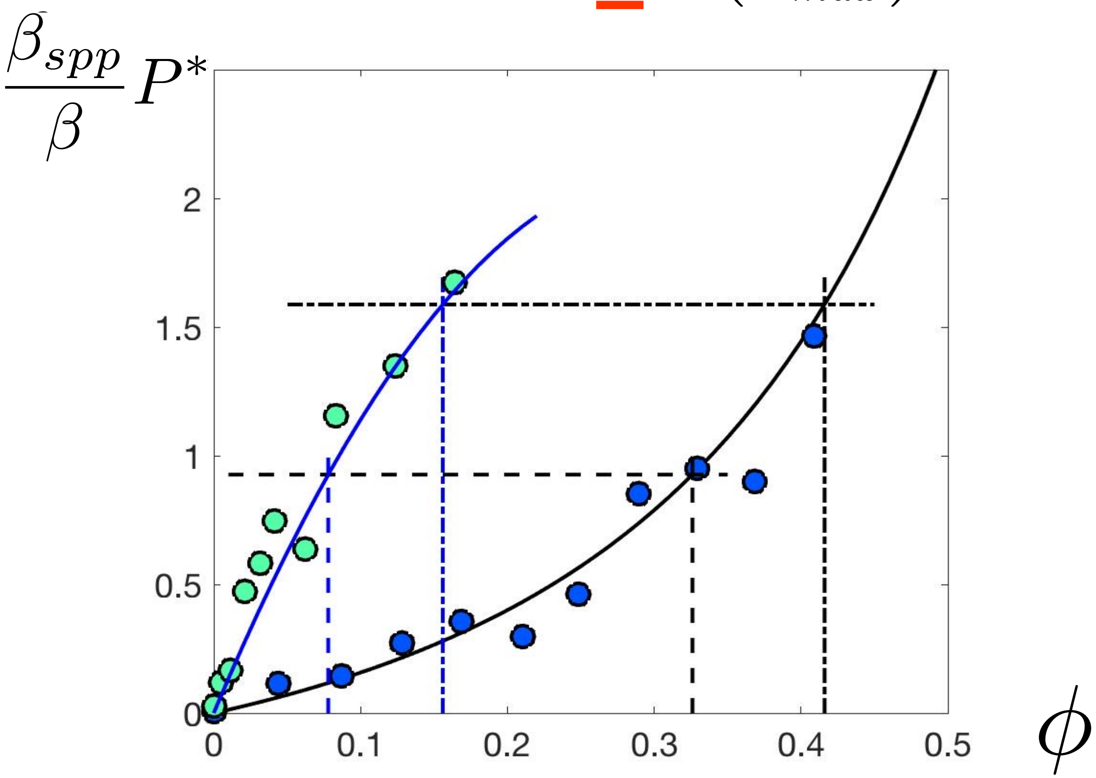




Φ_{iso}= 0.32 Φ_{spp}= 0.08

a : particle area

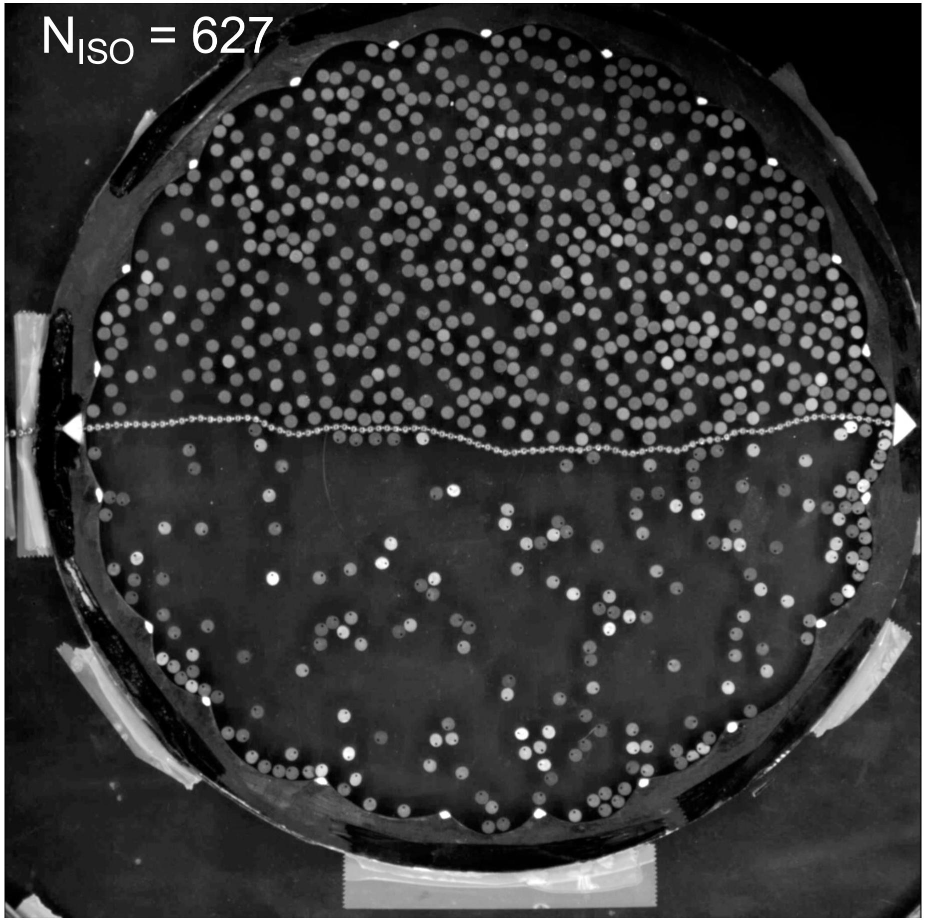
$$P^* = \beta P a = \frac{a}{\ell R}\mathcal{F}\left(\frac{\langle L\rangle}{L_{max}}\right)$$

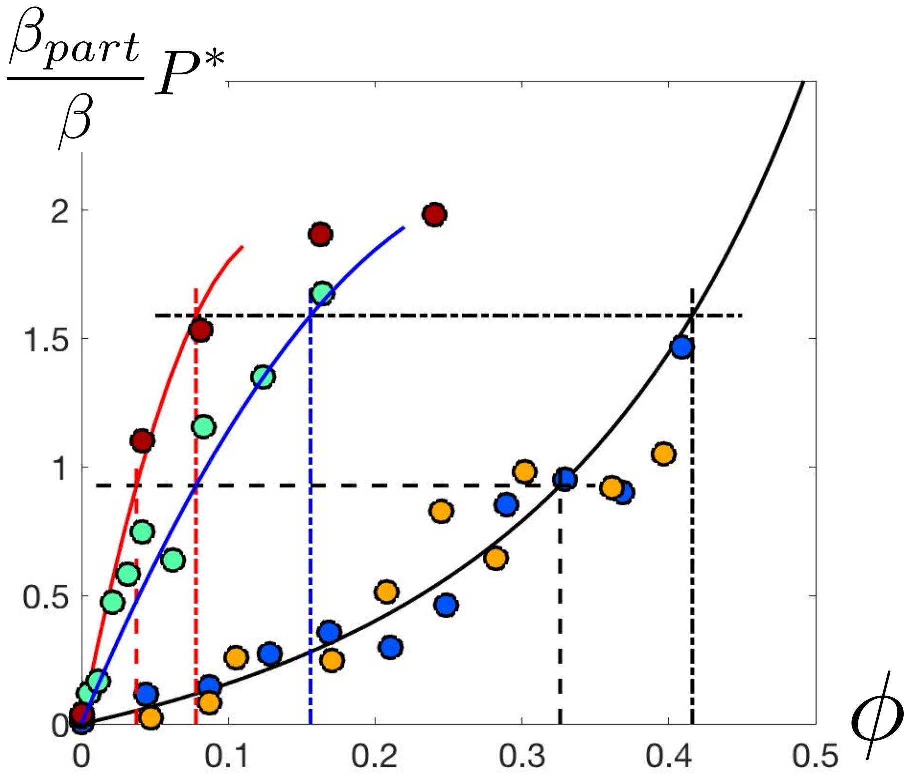


$$\Pi^*_{iso} = \frac{\beta}{\beta_{iso}}\phi\frac{1+\frac{1}{8}\phi^2}{(1-\phi)^2}$$

Equilibration for two different walls…

- Change the chain same total length L_{max} , but more units J = 147

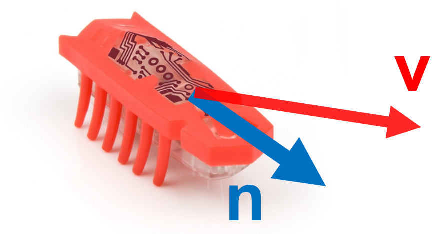




The mechanical pressure is not a state variable

The mechanical pressure is not a state variable

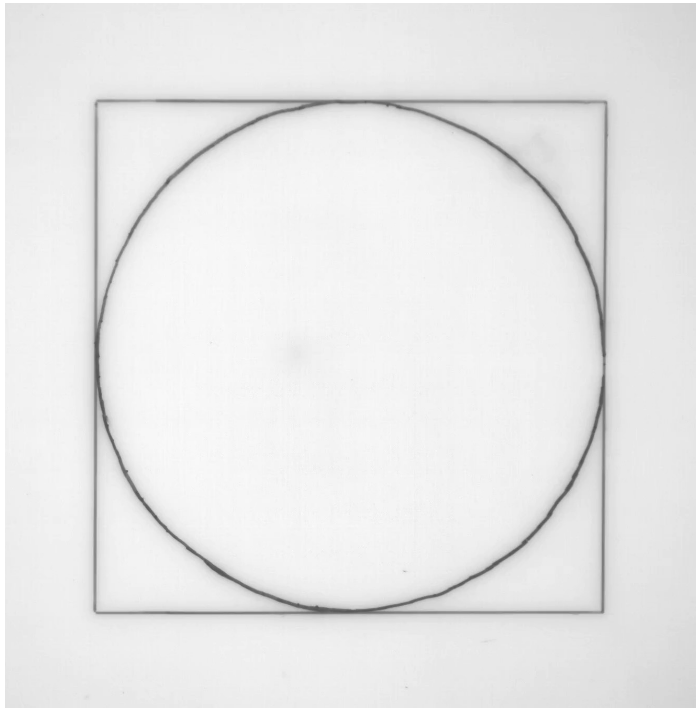
- Why ? the vibrated disk are a priori very similar to ABP, no EOS
- The reason is : active torque

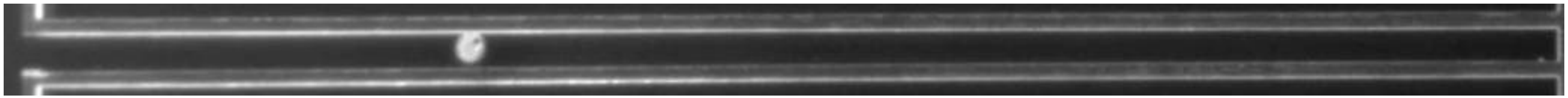


$$m\dot{\mathbf{v}} = F_0\hat{\mathbf{n}} - \gamma_t \mathbf{v} + \mathbf{F}_{ext}$$

$$J\dot{\omega} = \quad -\gamma_r \omega + \mathbf{\Gamma}_{ext} + \sqrt{2D}\xi \mathbf{n}_{\perp}$$

$$\Gamma_a = \zeta(\hat{\mathbf{n}} \times \mathbf{v}) \times \hat{\mathbf{n}}$$





Self-alignment

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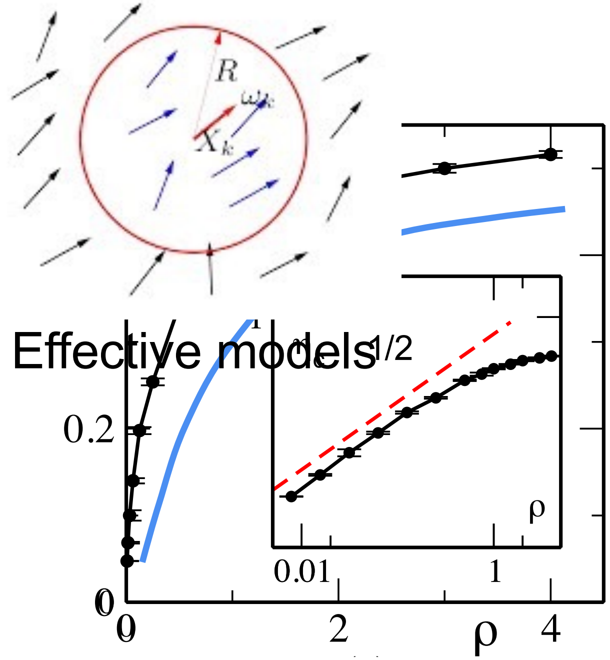
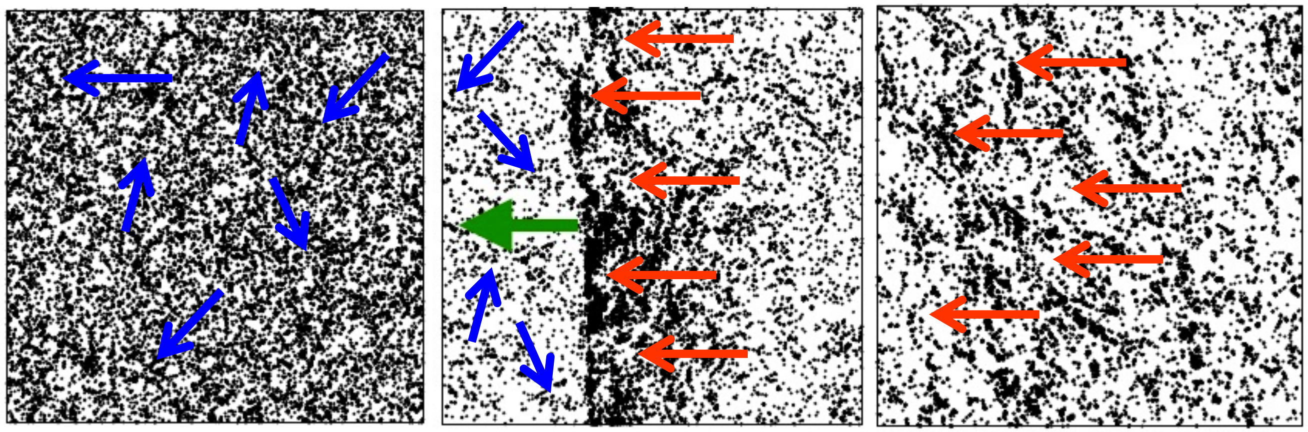
Transition to collective motion in Point Particles models

Vicseck model

- Over damped, self propelled Point Particles
  - Moving at velocity V_0 **n**
  - Alignment with neighbors (within some range)
  - + Noise

Effective models

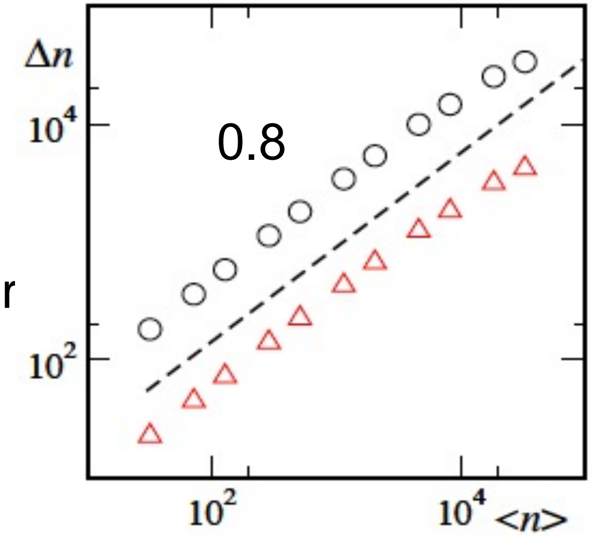




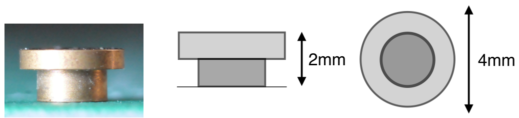
⇒ transition to collective motion is discontinuous

⇒ fast domain growth leading to high-density/high order solitary bands/sheets (2D/3D)

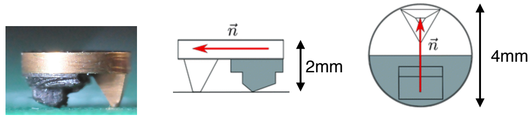
⇒ In the polar state: giant density fluctuations (splay modes)



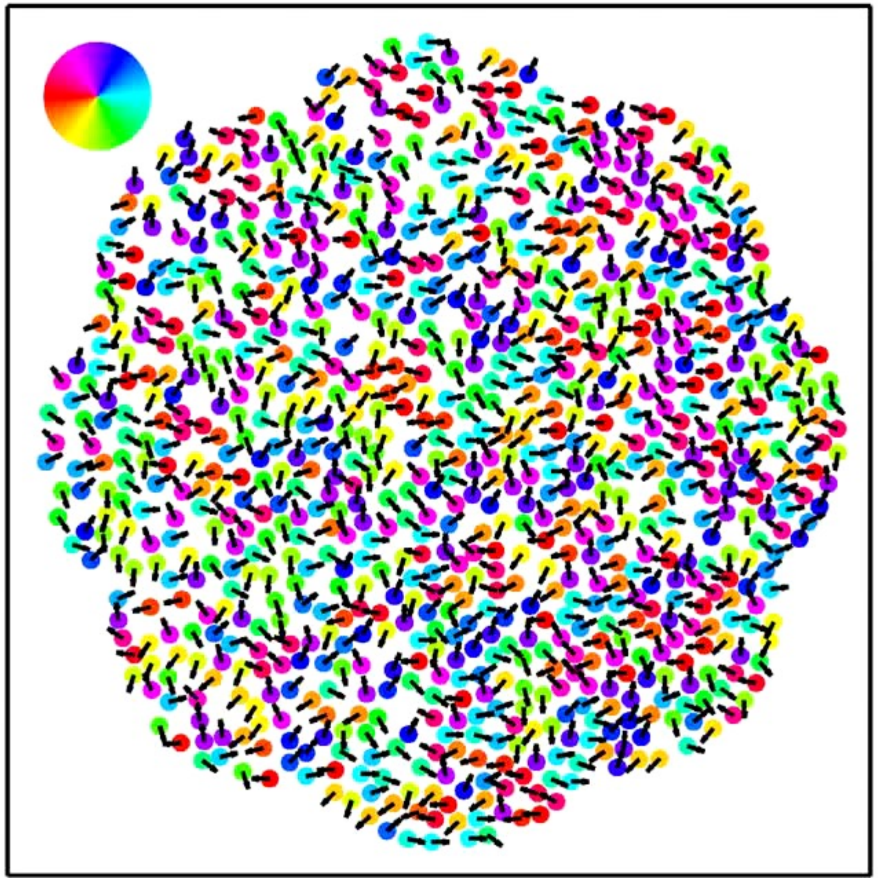
Experiment I :Vibrated grains

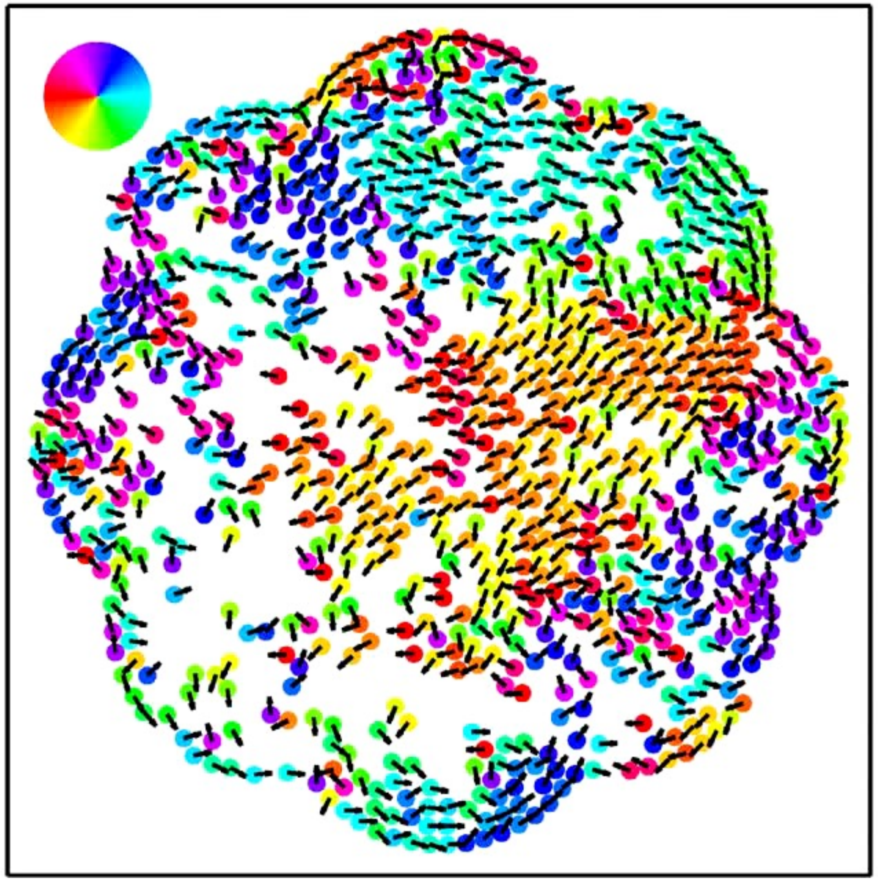


Isotropic particles



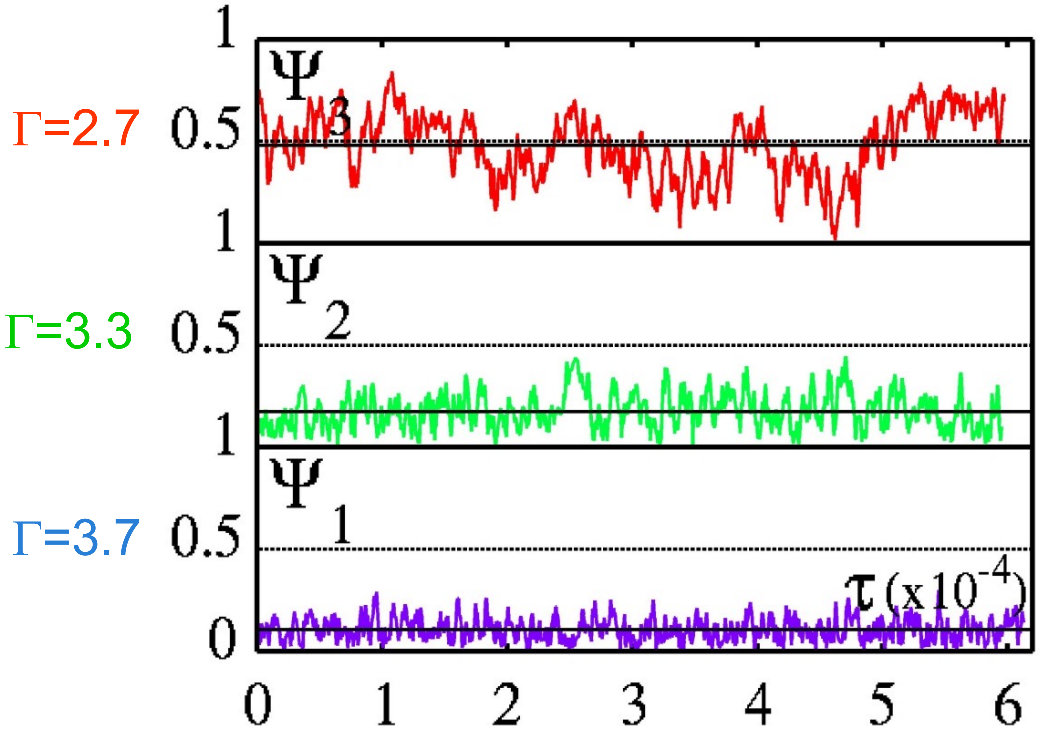
Polar particles

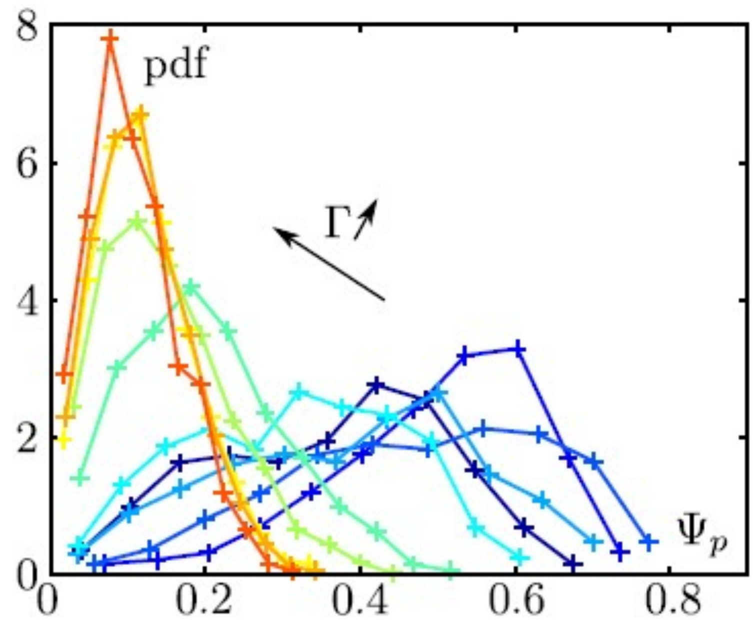


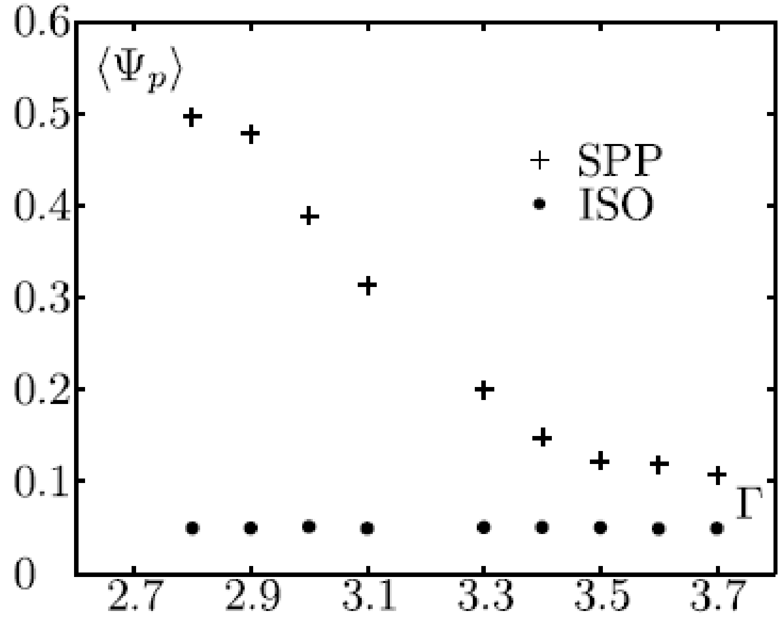


… collective motion and polar ordering

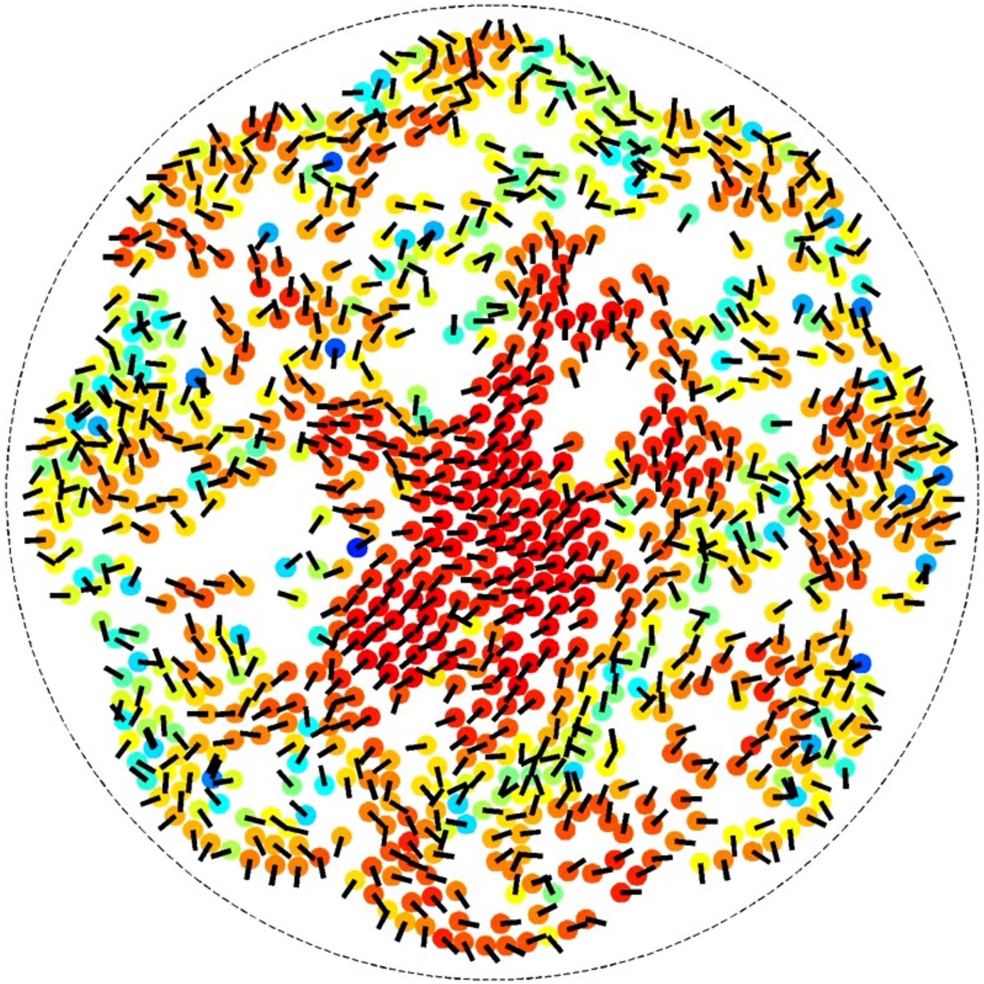
$$\Psi(t) = |\langle \vec{u}_i(t) \rangle|$$

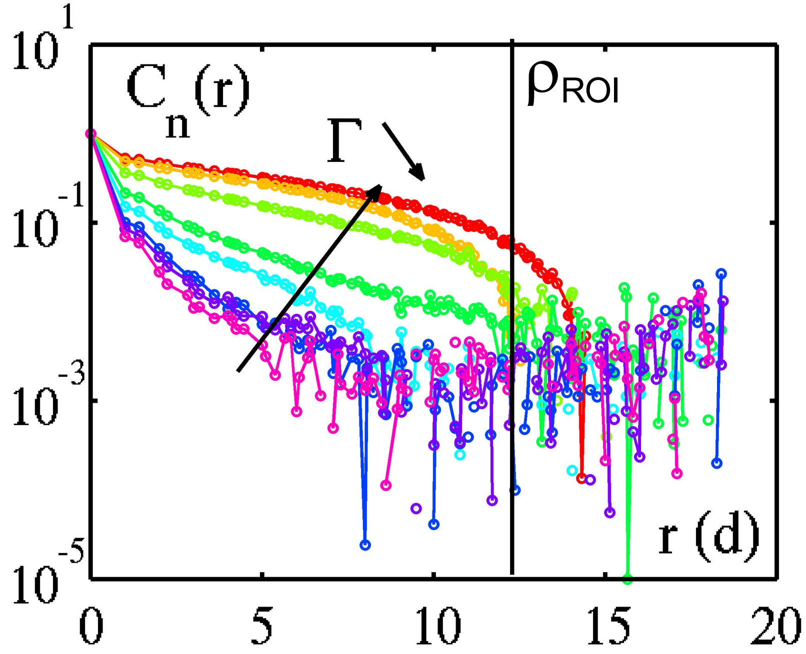


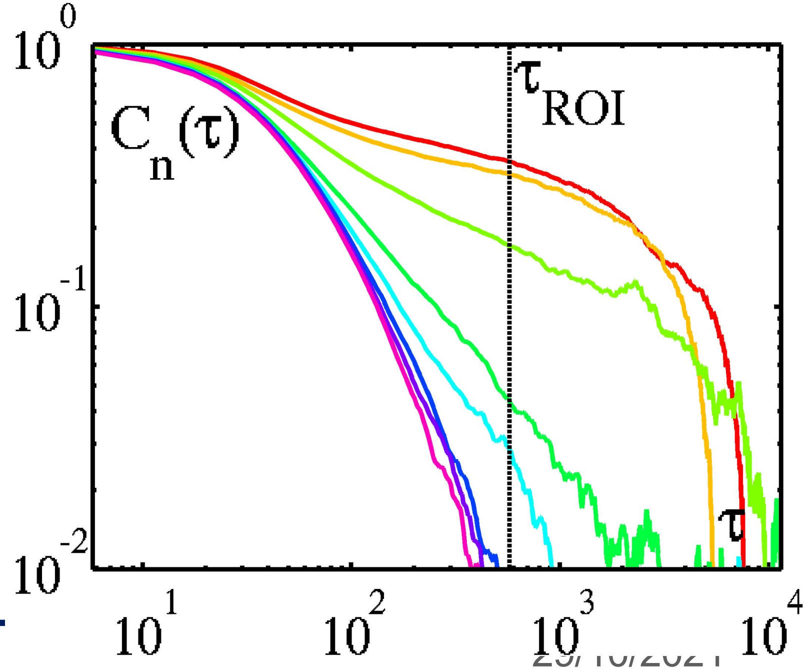




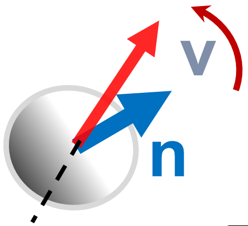
Collective motion color coded by local alignment







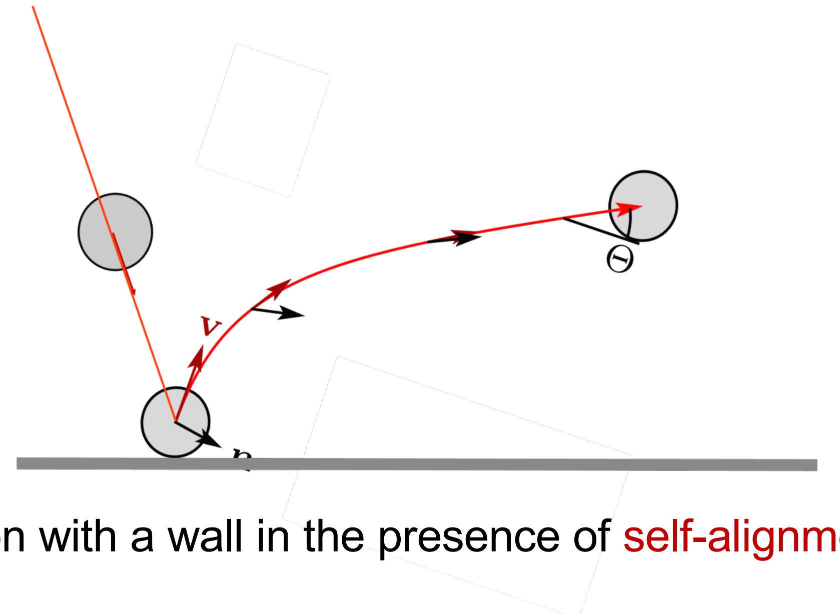
Why such collective motion for self propelled hard disks?



$$\tau_v \frac{\mathrm{d}}{\mathrm{d}t}\mathbf{v}_i = \hat{\mathbf{n}}_i - \mathbf{v}_i,$$ **+ collisions**

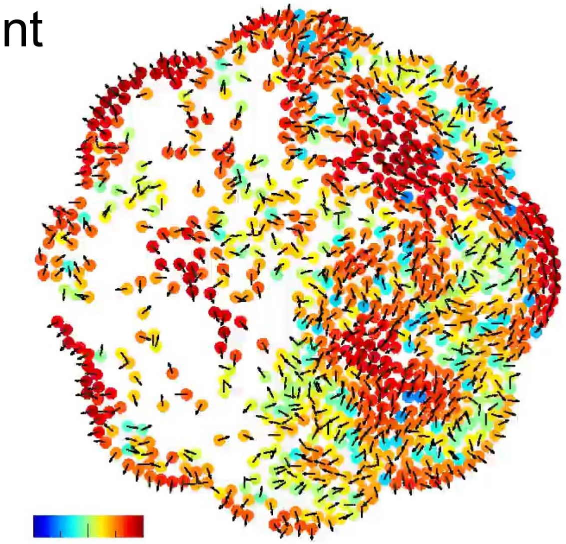
$$\tau_n \frac{\mathrm{d}}{\mathrm{d}t}\hat{\mathbf{n}}_i = (\hat{\mathbf{n}}_i \times \hat{\mathbf{v}}_i) \times \hat{\mathbf{n}}_i.$$

$\alpha = \tau_n / \tau_v$, persistence of $\hat{\mathbf{n}}_i$

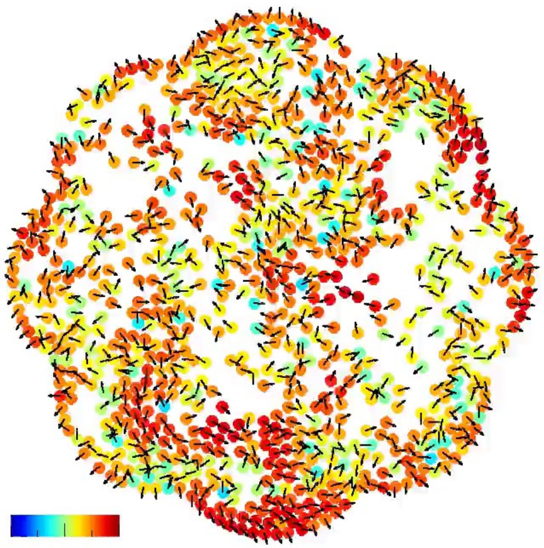


Collision with a wall in the presence of self-alignment

Experiment

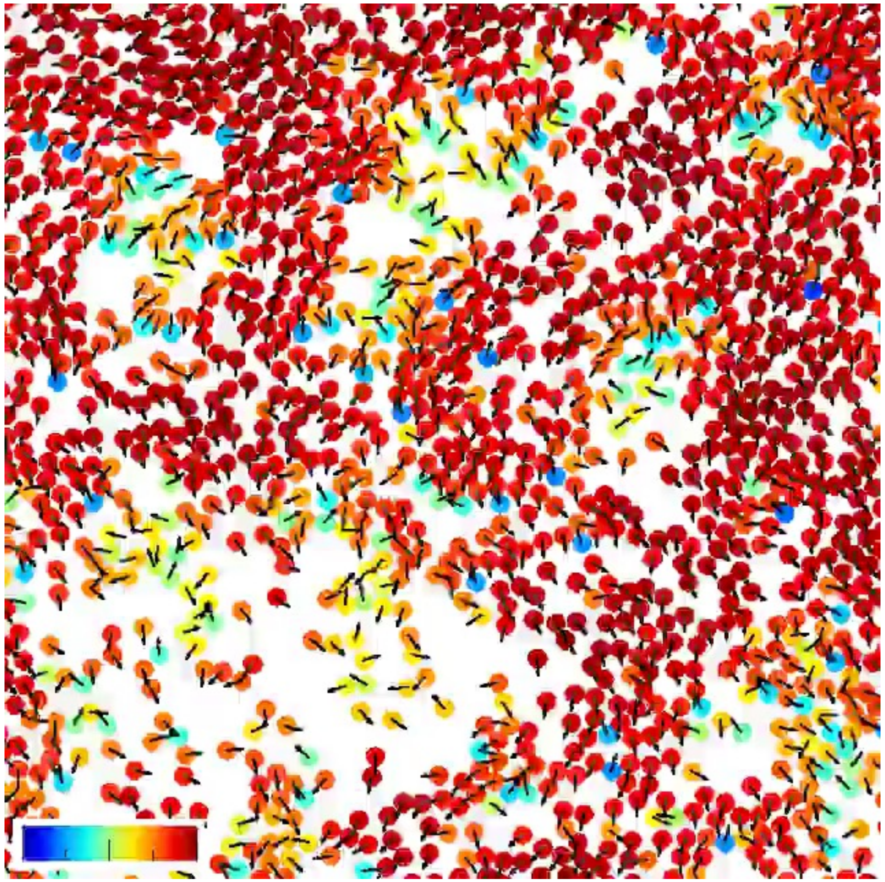


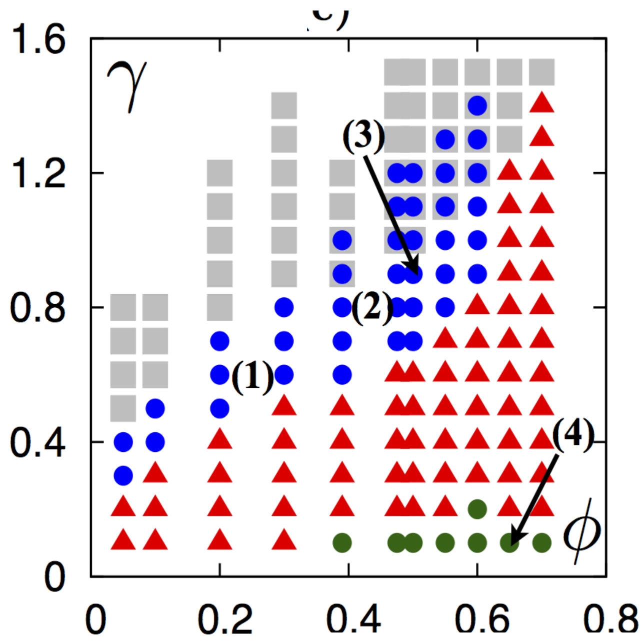
Model

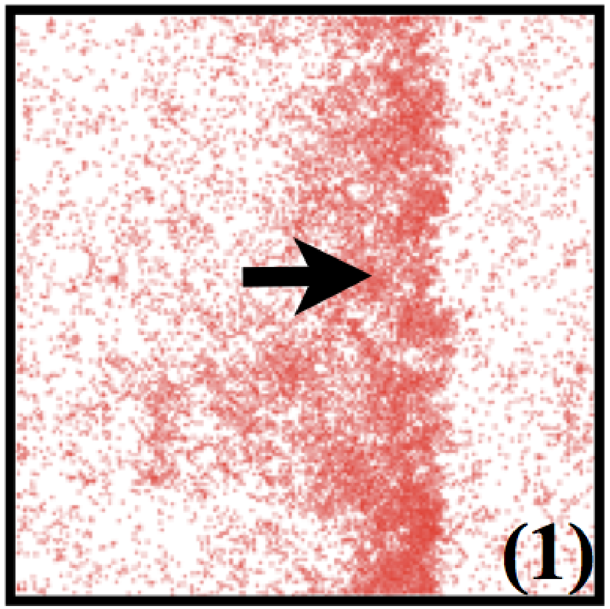


A true transition to collective motion

Periodic boundary conditions

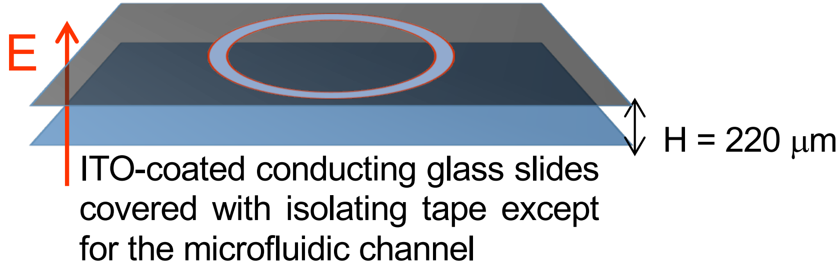


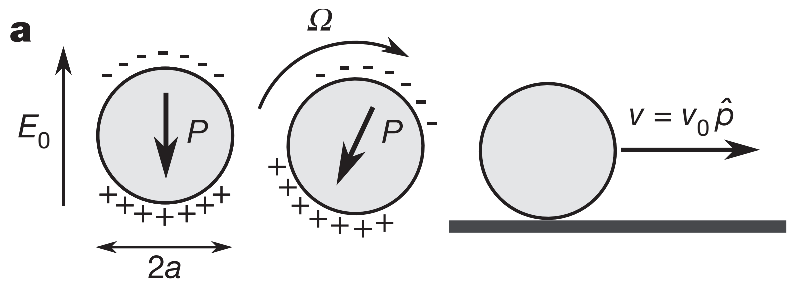




Experiment II : Rolling Colloids *(in coll. with D. Bartolo)*

- Goals :
  - A well controlled 2D experiment
  - Polar self propulsion
  - Interactions
    - Repulsion
    - Polar alignment

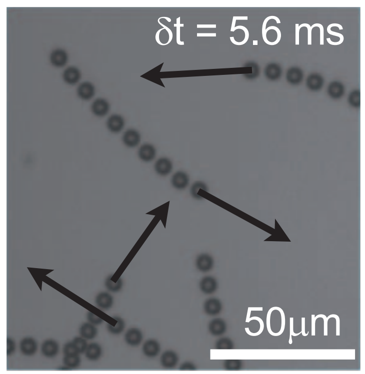


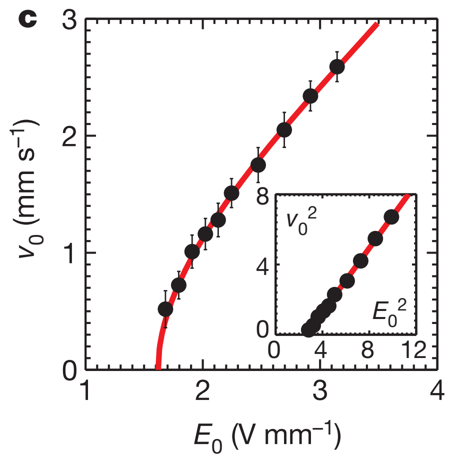


- Achieved with :
  - PMMA colloids (a=2.4 μm)
  - In AOT/hexadecane solution

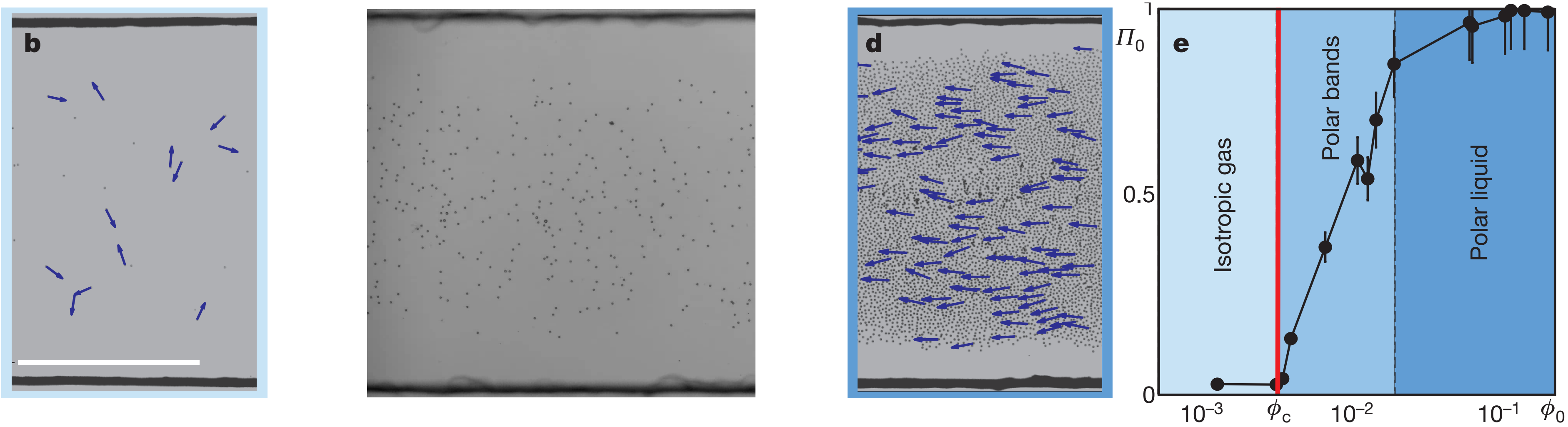
  - Dark field or Bright field microscopy
  - Acq between 70 and 900 i/s

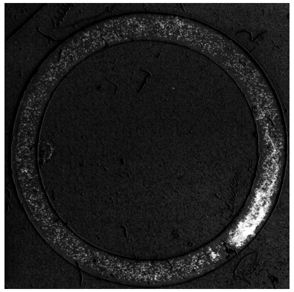
  - $V_0 \sim (E_0^2/E_Q^2-1)^{1/2} \sim 10^2$ and 10^3 d/s

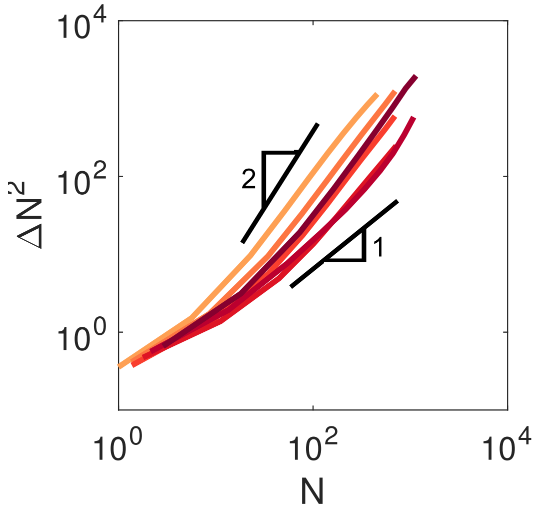




Transition to collective motion







- Increasing density
  - Isotropic -> polar bands -> homogeneous polar state
  - *Putative* giant density fluctuations

Interactions (electrostatics + far field hydrodynamics)

- Assuming pairwise interactions

$$\dot{\mathbf{r}}_i = v_0 \hat{\mathbf{p}}_\mathbf{i}$$

$$\dot{\theta}_i = \frac{1}{\tau}\frac{\partial}{\partial \theta_i}\sum_{j\neq i}\mathcal{H}_{\text{eff}}(\mathbf{r}_i - \mathbf{r}_j, \hat{\mathbf{p}}_i, \hat{\mathbf{p}}_j) + \sqrt{2D_r}\,\xi_i(t)$$

alignment repulsion dipolar LR

$$\mathcal{H}_{\text{eff}}(\mathbf{r}, \hat{\mathbf{p}}_i, \hat{\mathbf{p}}_j) = \boxed{A(r)\,\hat{\mathbf{p}}_j \cdot \hat{\mathbf{p}}_i} + \boxed{B(r)\,\hat{\mathbf{r}} \cdot \hat{\mathbf{p}}_i} + \boxed{C(r)\,\hat{\mathbf{p}}_j \cdot (2\hat{\mathbf{r}}\hat{\mathbf{r}} - \mathbf{I}) \cdot \hat{\mathbf{p}}_i}$$

hydrodynamics

electro-statics

$$A(r) = \boxed{3\tilde{\mu}_s \frac{a^3}{r^3}\Theta(r)} + 9\left(\frac{\mu_\perp}{\mu_r} - 1\right)\left(\chi^\infty + \frac{1}{2}\right)\left(1 - \frac{E_Q^2}{E_0^2}\right)\frac{a^5}{r^5}\Theta(r)$$

$$B(r) = 6\left(\frac{\mu_\perp}{\mu_r} - 1\right)\sqrt{\frac{E_0^2}{E_Q^2} - 1}\left[\left(\chi^\infty + \frac{1}{2}\right)\frac{E_Q^2}{E_0^2} - \chi^\infty\right]\frac{a^4}{r^4}\Theta(r)$$

$$C(r) = \boxed{6\tilde{\mu}_s \frac{a}{H}\frac{a^2}{r^2}} + 3\tilde{\mu}_s \frac{a^3}{r^3}\Theta(r) + 15\left(\frac{\mu_\perp}{\mu_r} - 1\right)\left(\chi^\infty + \frac{1}{2}\right)\left(1 - \frac{E_Q^2}{E_0^2}\right)\frac{a^5}{r^5}\Theta(r)$$

Kinetic theory

- From N Langevin equation to Fokker Planck for the N part distribution

$$\frac{\partial \Psi^{(N)}}{\partial t} + \sum_i \nabla_i \cdot \left(v_0 \hat{\mathbf{p}}_i \Psi^{(N)} \right) + \sum_i \frac{\partial}{\partial \theta_i} \left(\frac{1}{\tau} \sum_{j \neq i} \frac{\partial \mathcal{H}_{\text{eff}}(\mathbf{r}_i - \mathbf{r}_j, \theta_i, \theta_j)}{\partial \theta_i} \Psi^{(N)} \right) - D_r \sum_i \frac{\partial^2}{\partial \theta_i^2} \Psi^{(N)} = 0$$

- Integrating out N-1 particles :

$$\partial_t \Psi^{(1)} + v_0\, \hat{\mathbf{p}} \cdot \nabla \Psi^{(1)} + \frac{1}{\tau} \partial_\theta \int d^2\mathbf{r}' \mathrm{d}\theta' \, \frac{\partial \mathcal{H}_{\text{eff}}(\mathbf{r} - \mathbf{r}', \theta, \theta')}{\partial \theta} \Psi^{(2)}(\mathbf{r}, \mathbf{r}', \theta, \theta', t) - D_r\, \partial_\theta^2 \Psi^{(1)} = 0$$

- Molecular Chaos hypothesis + exclusion volume

$$\Psi^{(2)}(\mathbf{r}, \mathbf{r}', \theta, \theta', t) = \begin{cases} 0 & \textbf{if } |\mathbf{r} - \mathbf{r}'| < 2a \\ \Psi^{(1)}(\mathbf{r}, \theta, t) \Psi^{(1)}(\mathbf{r}', \theta', t) & \textbf{if } |\mathbf{r} - \mathbf{r}'| \geq 2a \end{cases}$$

- A close integro-differential equation for $\Psi^{(1)}(r, \theta, t)$

$$\boxed{\partial_t \Psi - v_0\, \hat{\mathbf{p}} \cdot \nabla \Psi = \frac{1}{\tau} \partial_\phi \int_{|\mathbf{r} - \mathbf{r}'| \geq d} \Psi(\mathbf{r}, \phi, t)\, F(\mathbf{r} - \mathbf{r}', \phi, \phi')\, \Psi(\mathbf{r}', \phi', t)\ \mathrm{d}^2\mathbf{r}' \mathrm{d}\phi' + D_r \frac{\partial^2}{\partial \phi^2} \Psi}$$

Hydrodynamics Theory

- Defining the hydrodynamics fields

$$\phi(\mathbf{r},t) \equiv \frac{1}{\pi a^2}\int d\theta\ \Psi^{(1)}(\mathbf{r},\theta,t)$$
$$\mathbf{\Pi}(\mathbf{r},t) \equiv \frac{\pi a^2}{\phi}\int d\theta\ \hat{\mathbf{p}}\Psi^{(1)}(\mathbf{r},\theta,t)$$
$$\mathbf{Q}(\mathbf{r},t) \equiv \frac{\pi a^2}{\phi}\int d\theta\ \left(\hat{\mathbf{p}}\hat{\mathbf{p}} - \frac{1}{2}\mathbf{I}\right)\Psi^{(1)}(\mathbf{r},\theta,t)$$

- Hydrodynamics equations

$$\partial_t\,\phi + v_0\nabla\cdot(\phi\mathbf{\Pi}) = 0$$
$$\partial_t\mathbf{\Pi}(\mathbf{r},t) = \mathcal{F}_\Pi(\Phi,\mathbf{\Pi},\mathbf{Q})$$
$$\partial_t\mathbf{Q}(\mathbf{r},t) = \mathcal{F}_Q(\Phi,\mathbf{\Pi},\mathbf{Q},\text{higher order moments})$$

- Closure relations
  - Close to the isotropic solution $\Psi^{(1)}(r,\theta,t) \propto \frac{1}{2\pi}(1+2|\mathbf{\Pi}|cos(\theta))$

$$\tau\partial_t(\phi\mathbf{\Pi}) + \frac{3v_0\alpha}{8D_r}(\phi\mathbf{\Pi}\cdot\nabla)\,\phi\mathbf{\Pi} = \left[\alpha\,\phi - \tau D_r - \frac{\alpha^2}{2\tau D_r}(\phi^2\Pi^2)\right]\phi\mathbf{\Pi} + \kappa\phi\,\mathbf{M}*\phi\mathbf{\Pi} - \frac{1}{2}(\tau v_0 + a\beta\phi)\,\nabla\phi$$
$$- \frac{5v_0\alpha}{8D_r}(\nabla\cdot\phi\mathbf{\Pi})\,\phi\mathbf{\Pi} + \frac{5v_0\alpha}{16D_r}\nabla(\phi^2\Pi^2) + \frac{\alpha\beta}{2\tau D_r}a(\nabla\phi\cdot\phi\mathbf{\Pi})\,\phi\mathbf{\Pi} + \mathcal{O}($$

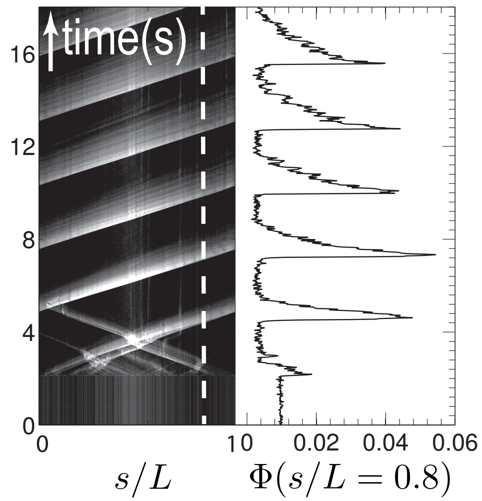
  - Close to the polar solution $\Psi^{(1)}(r,\theta,t) \simeq \text{Gaussian around}\ \bar{\theta}$

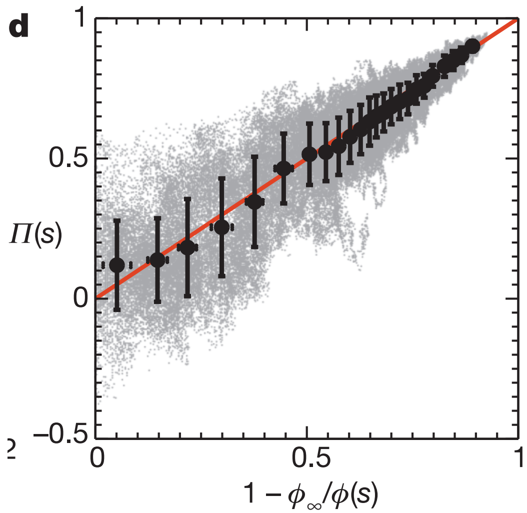
$$\tau\partial_t\mathbf{\Pi} + \tau v_0(\mathbf{\Pi}\cdot\nabla)\mathbf{\Pi} = \left[2a^2(\beta+\gamma)(1-\Pi^2)\rho - \tau D_r\right]\mathbf{\Pi} - 2a^3\alpha(\mathbb{1} - \mathbf{\Pi}\,\mathbf{\Pi})\cdot\nabla\rho$$
$$- 2a^2\kappa(\mathbb{1} - \mathbf{\Pi}\,\mathbf{\Pi})\mathbf{M}\cdot(\rho\mathbf{\Pi}) + \mathcal{O}(\nabla^2)$$

Hydrodynamics applications (I)

- Near onset :
  - Homogeneous solution: $$\Pi_0(\phi_0) = \begin{cases} \sqrt{2\frac{\phi_c}{\phi_0}\left(1-\frac{\phi_c}{\phi_0}\right)} & \text{if } \phi_0 > \phi_c \\ 0 & \text{if } \phi_0 \le \phi_c \end{cases}$$
  - Linear stability analysis => homogeneous solution is unstable

- Steady propagating solutions : $$\Pi(s) = \frac{c_{\text{band}}}{v_0}\left(1-\frac{\phi_\infty}{\phi(s)}\right)$$



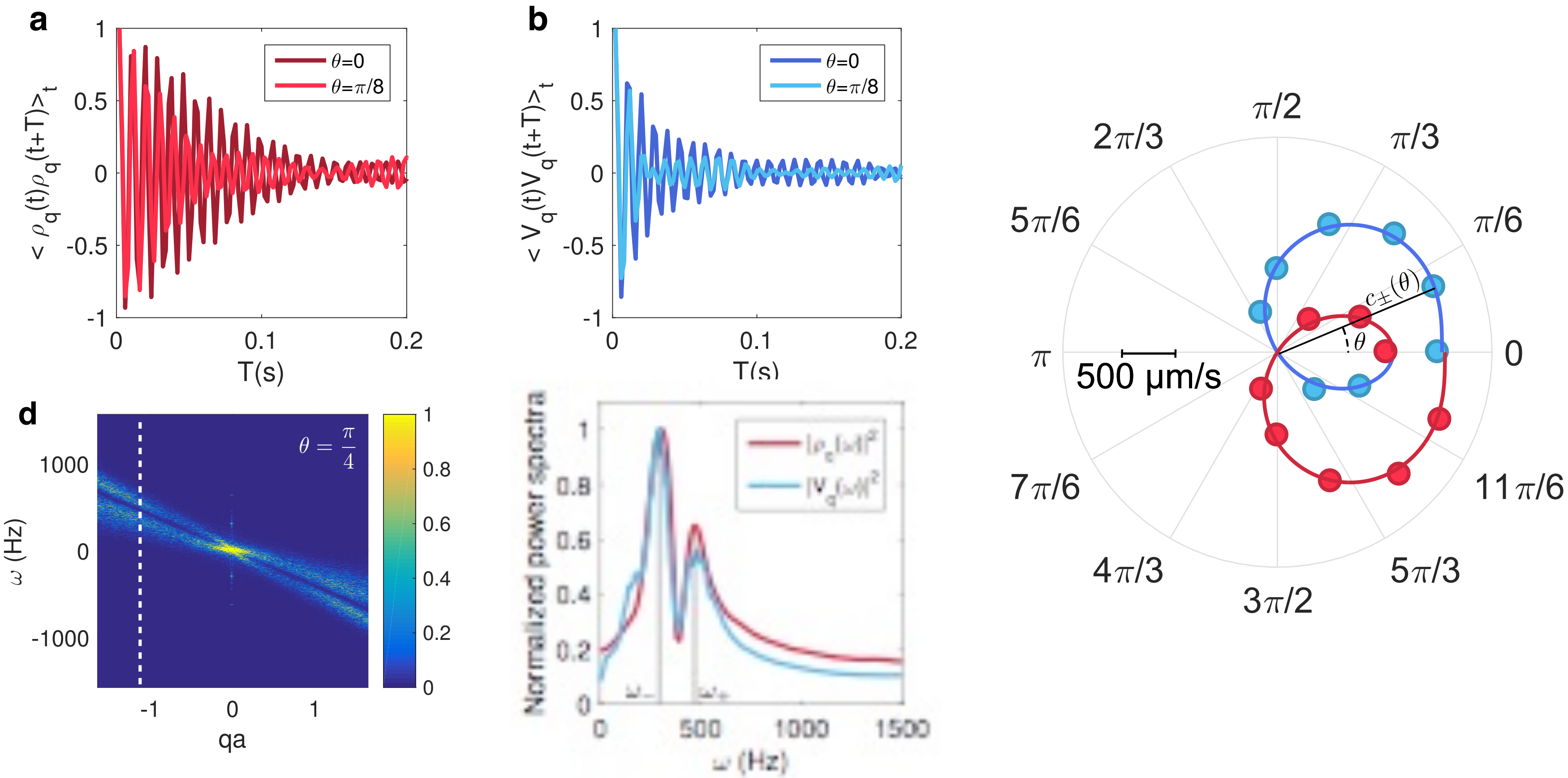


Hydrodynamics theory (ii)

- In the polar phase : sound propagation

D Geyer, A Morin & D Bartolo
Nature Materials **17**, 789–793 (2018)

$$2c_{\pm}(\theta) = (1+\lambda_1)\,u_0\cos\theta \pm \sqrt{(\lambda_1-1)^2\,u_0^2\cos^2\theta + 4\sigma\rho_0\sin^2\theta}.$$



Summary for rolling colloids

- Constant velocity
- Explicit Alignment interactions (electrostatic and hydrodynamics) at low enough density to avoid hard core interactions => point like.

=> the perfect system for realizing the Vicsek scenario

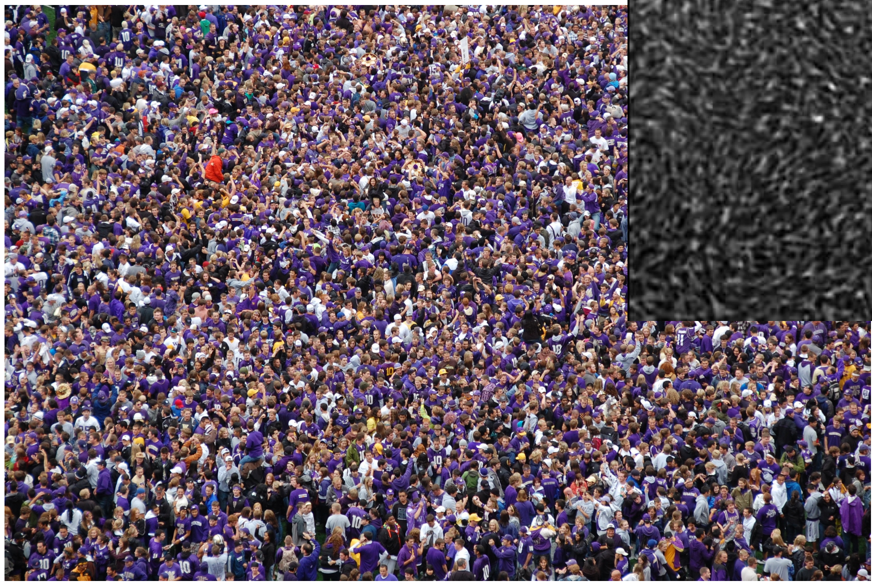
- Indeed observed
  - First order transition to collective motion
  - Polar bands
  - True Long Range Order polar motion
- Sound waves in the polar phase
  - An excellent confirmation of the linear hydrodynamics theory
- Giant density fluctuations
  - Present but impossible to validate exponent

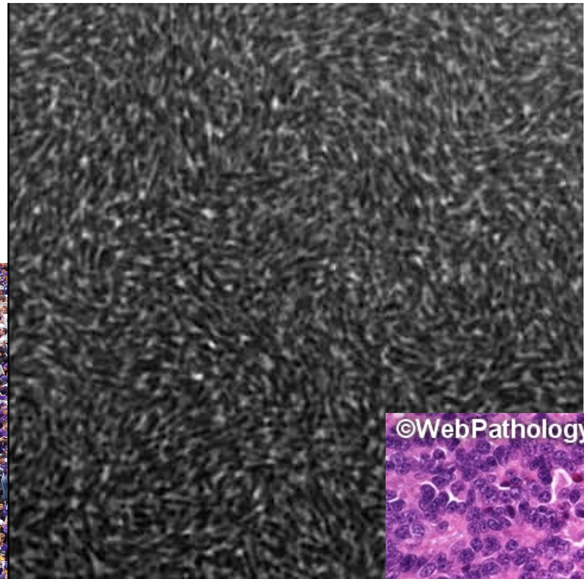
Outline: from active liquids to active solids

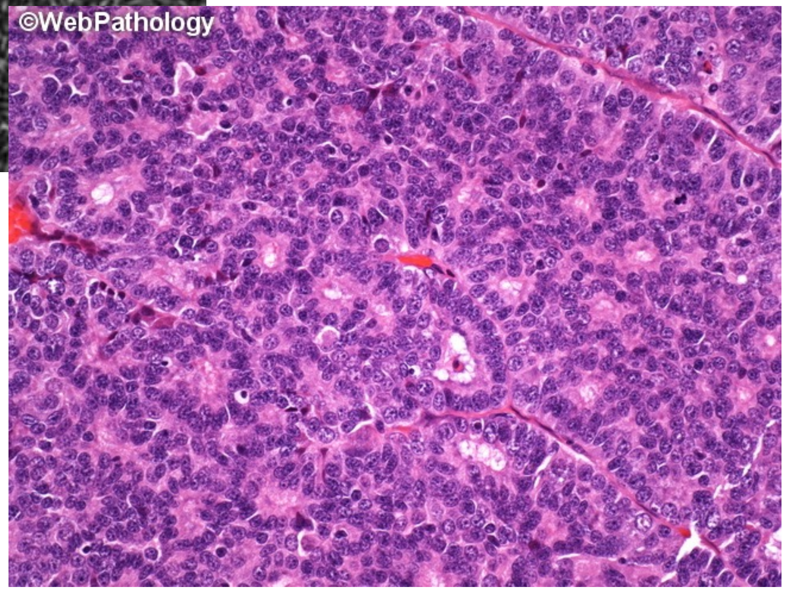
- Active fluids : a brief overview with a focus on collective motion
  - mechanical pressure is not a state variable
  - liquid-gas phase separation takes place in purely repulsive systems
  - macroscopic flows emerge in the absence of external gradient

- Active solids :
  - spontaneous flows also take place in crystalline structure
  - selective & collective actuation emerges in linear elastic systems

Increasing Packing Fraction might be relevant…

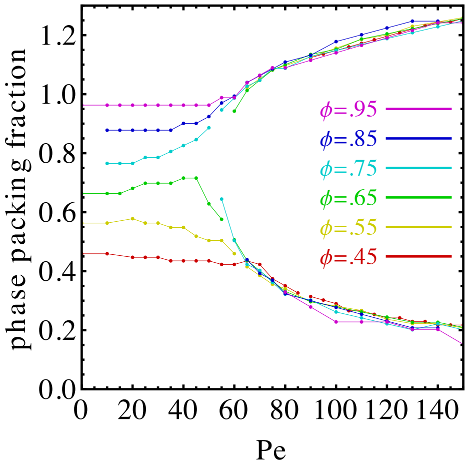


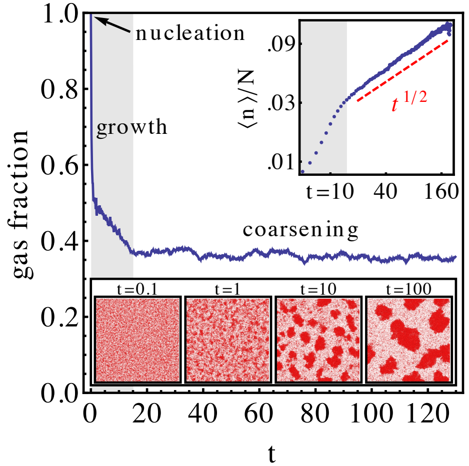




Possible situations & questions

- Crowding effects
  - Slowing down => MIPS

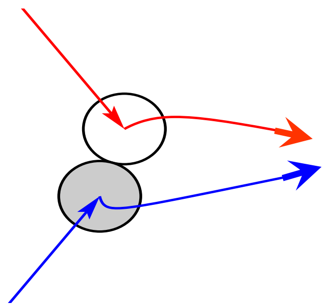




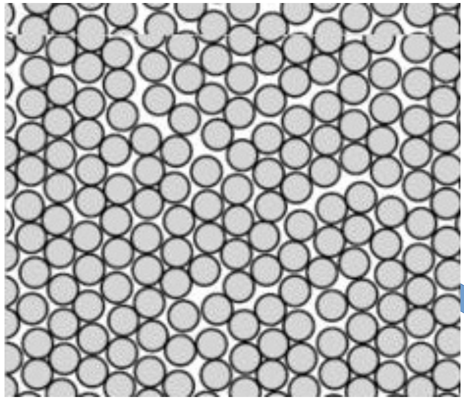
  - Alignment could suppress the slowing down => avoided MIPS

- Structural ordering
  - Does crystallization takes place or do active stresses prevent it?
  - Alignment could reduce the active stresses => promote the crystal

- Today : One specific system -> Self Propelled Hard Disks
  - Does spontaneous alignment survive?

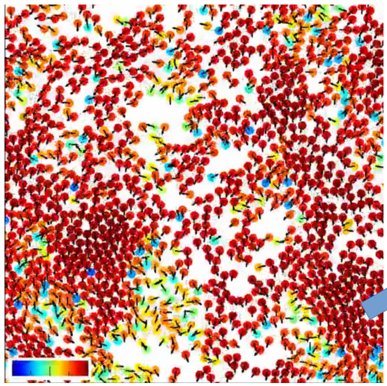


Phase space : what shall we expect?

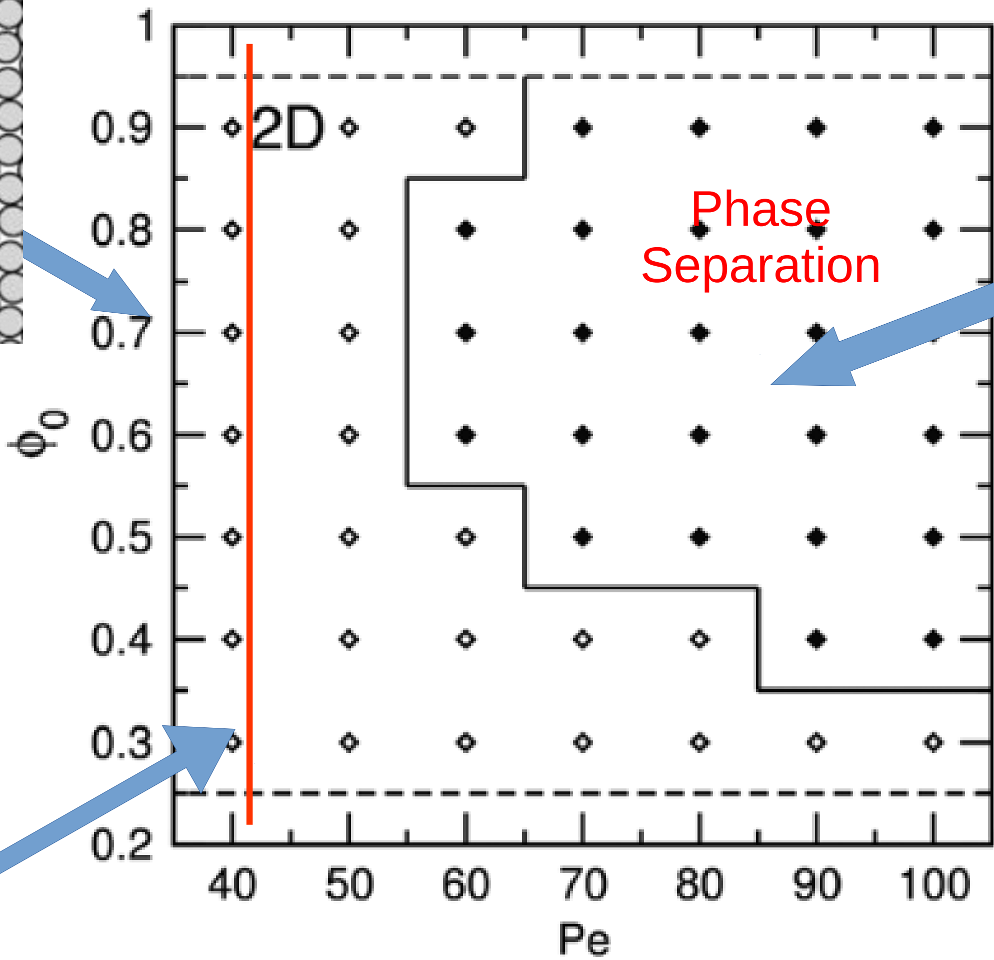


Equilibrium
Crystallization

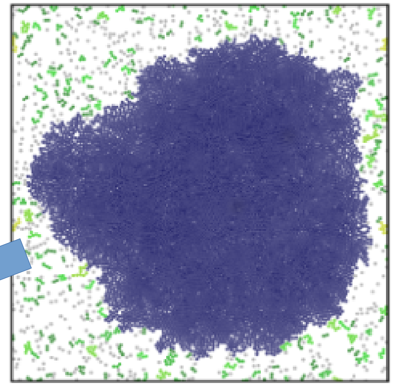
Pe << 1



Collective motion
Pe ~ 1-10

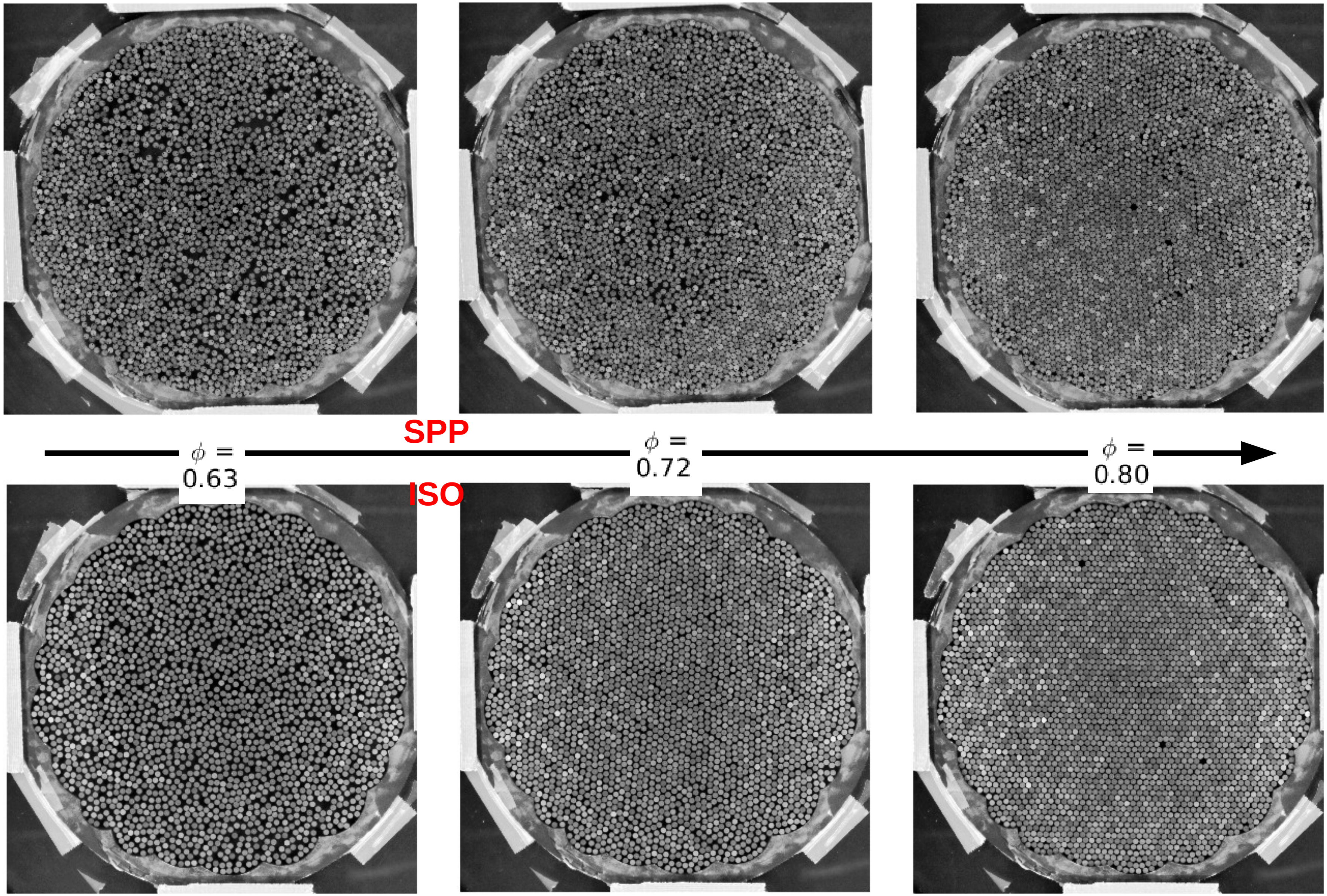


Persistence length / d

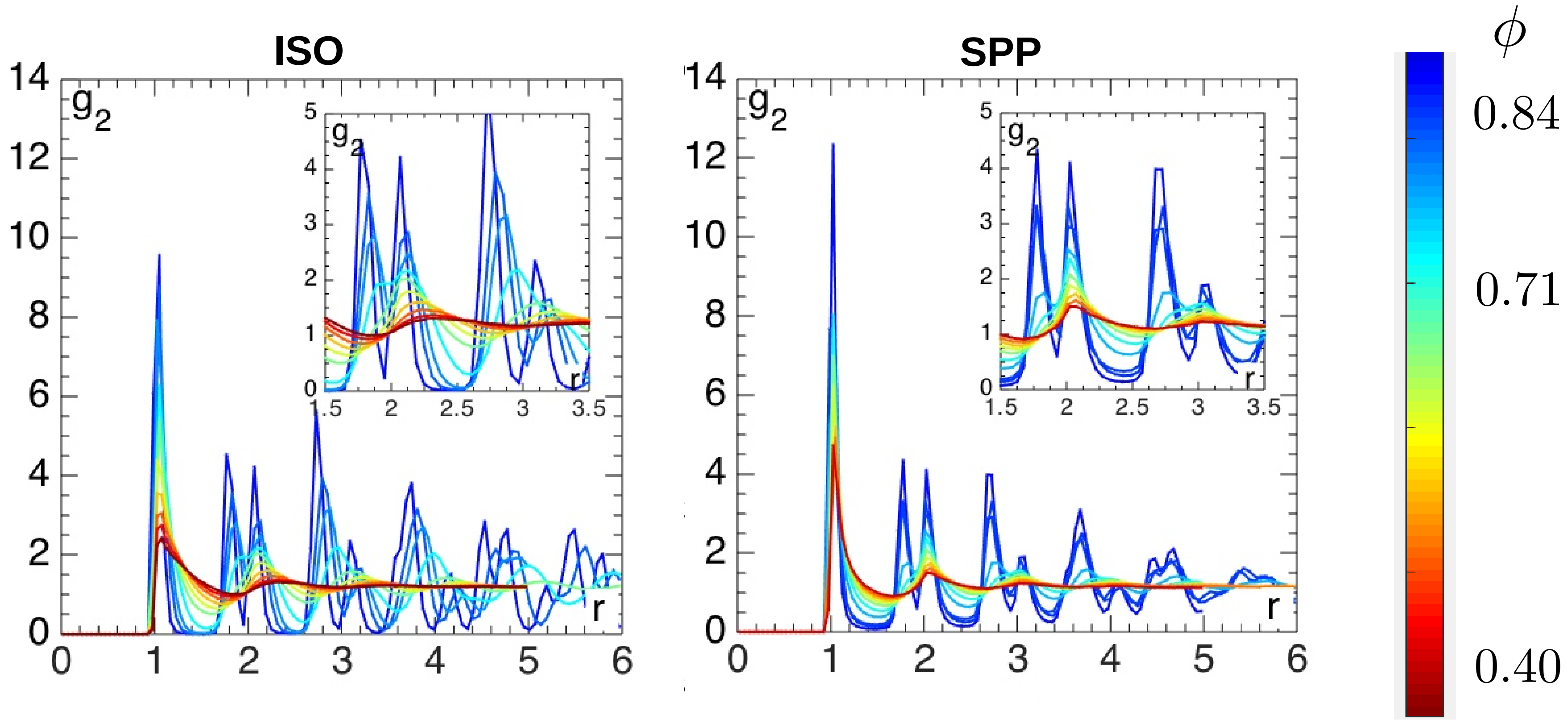


MIPS

At first sight …



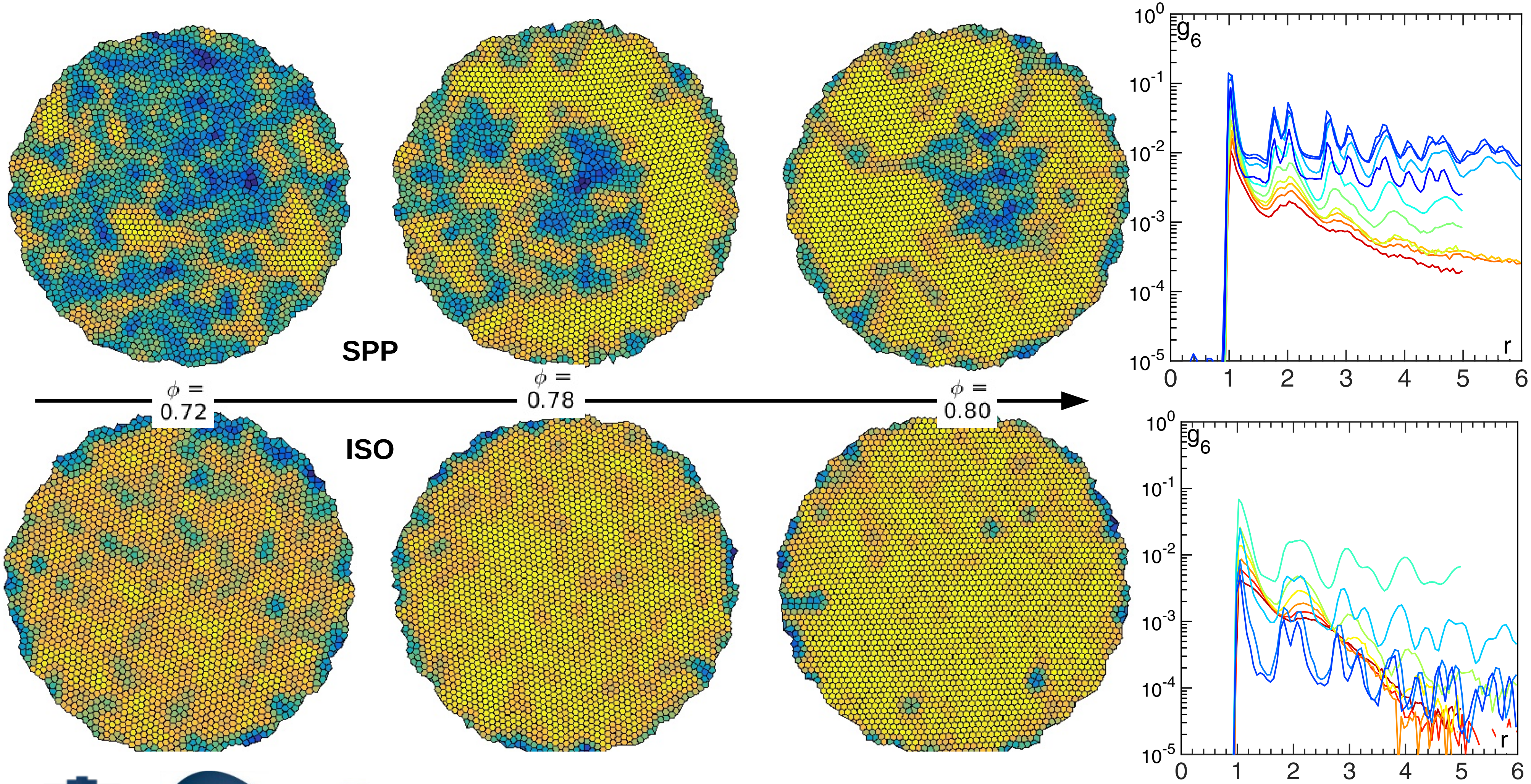
Structural properties : pair correlation function



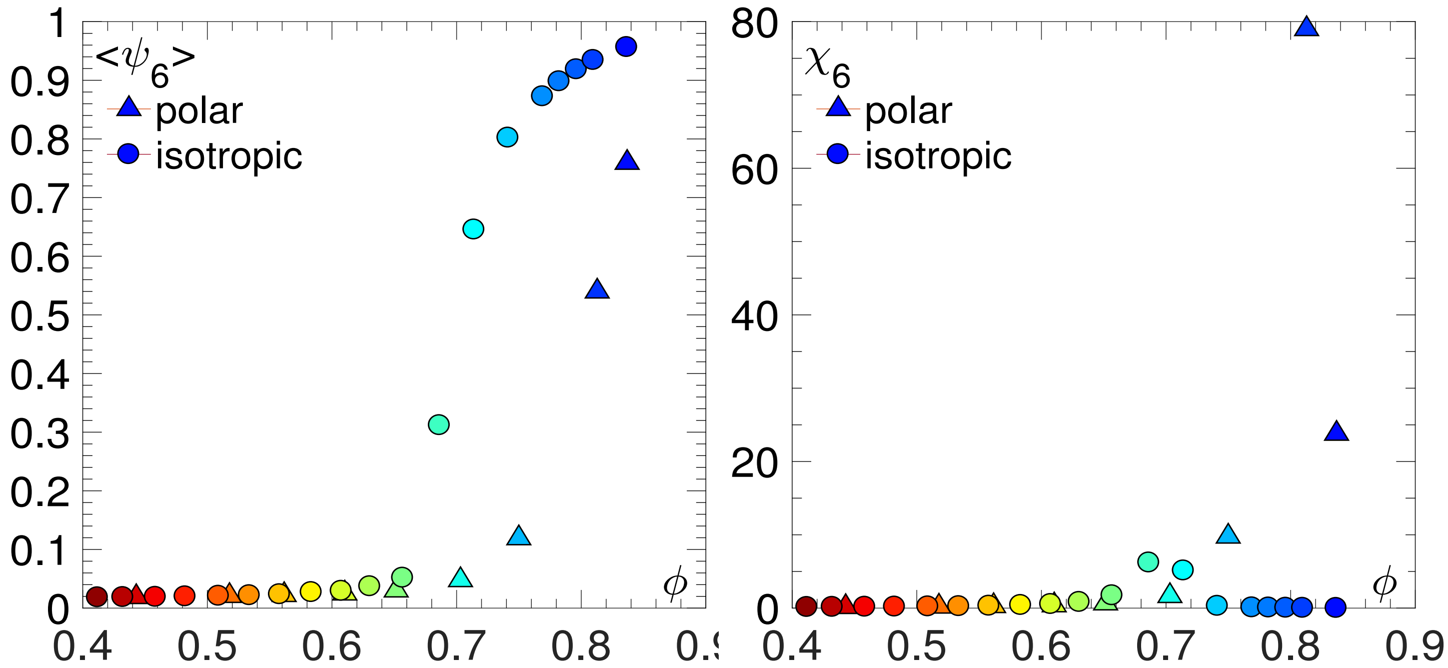
- In both cases order emerges for $\phi = 0.72$
- However in the active case
  - Correlation length is smaller
  - More importantly : order sets in with almost a close packed structure!

Structural properties : orientational order

$$\psi_6^p = \left[\frac{1}{n_p}\sum_{\langle pq\rangle} \exp(6i\theta_{pq})\right]$$

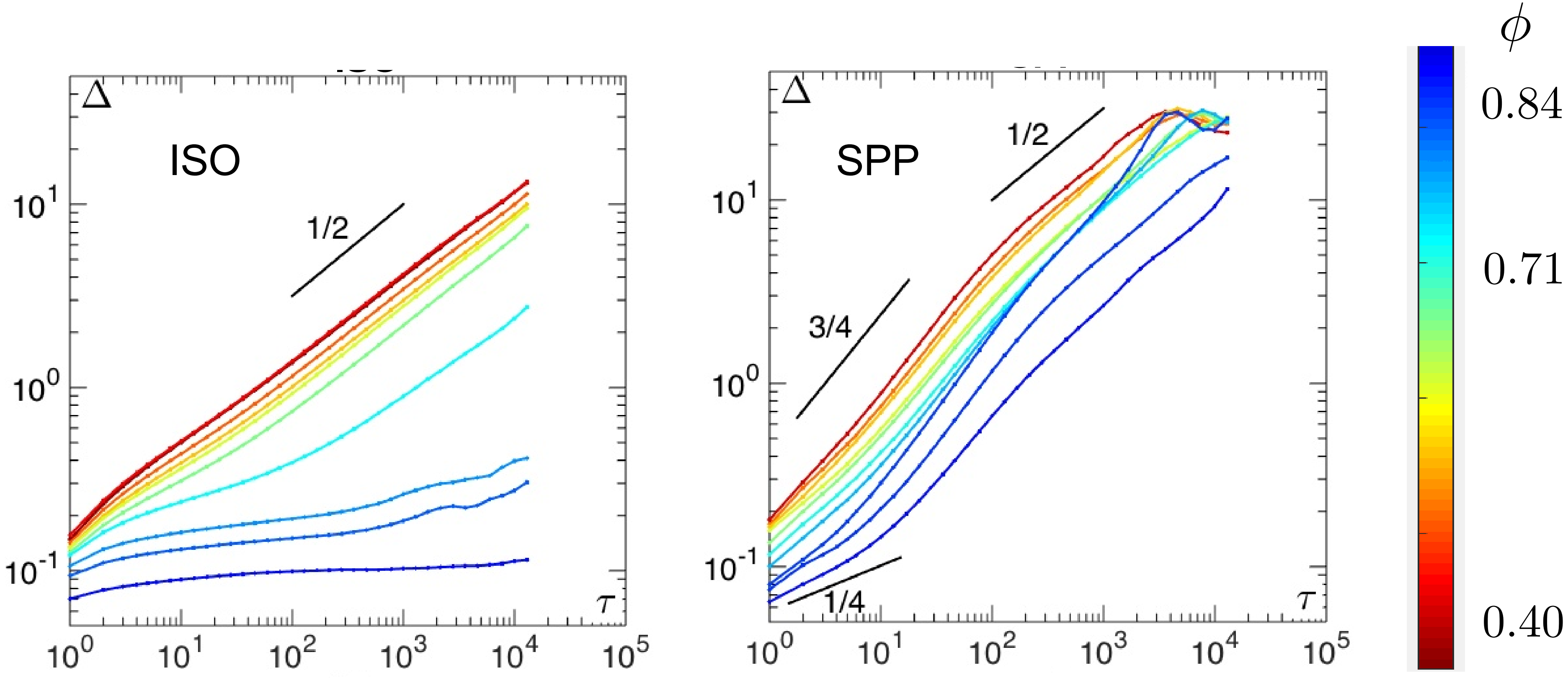


Structural properties : orientational order



- The transition to an ordered phase is delayed to much higher density
- The order of the transition is unclear : phase coexistence?

Dynamics : Mean Square Displacement

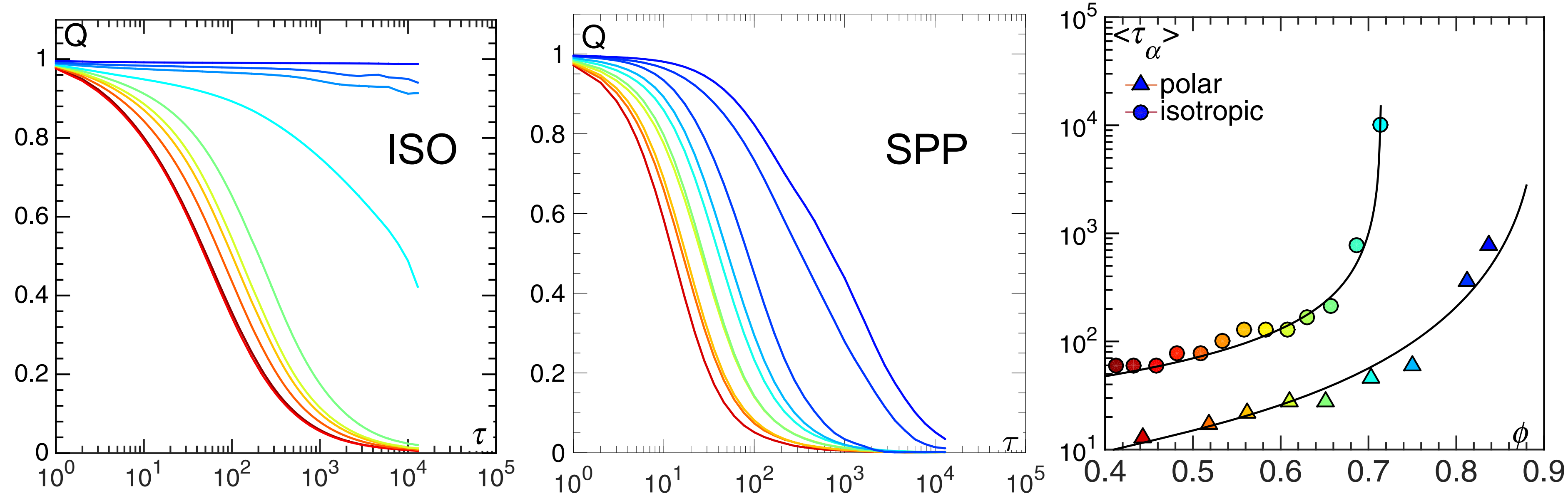


- The abrupt caging observed at equilibrium never takes place!
- The dynamics remains super-diffusive at intermediate timescales

=> dynamics and structure fully decouple

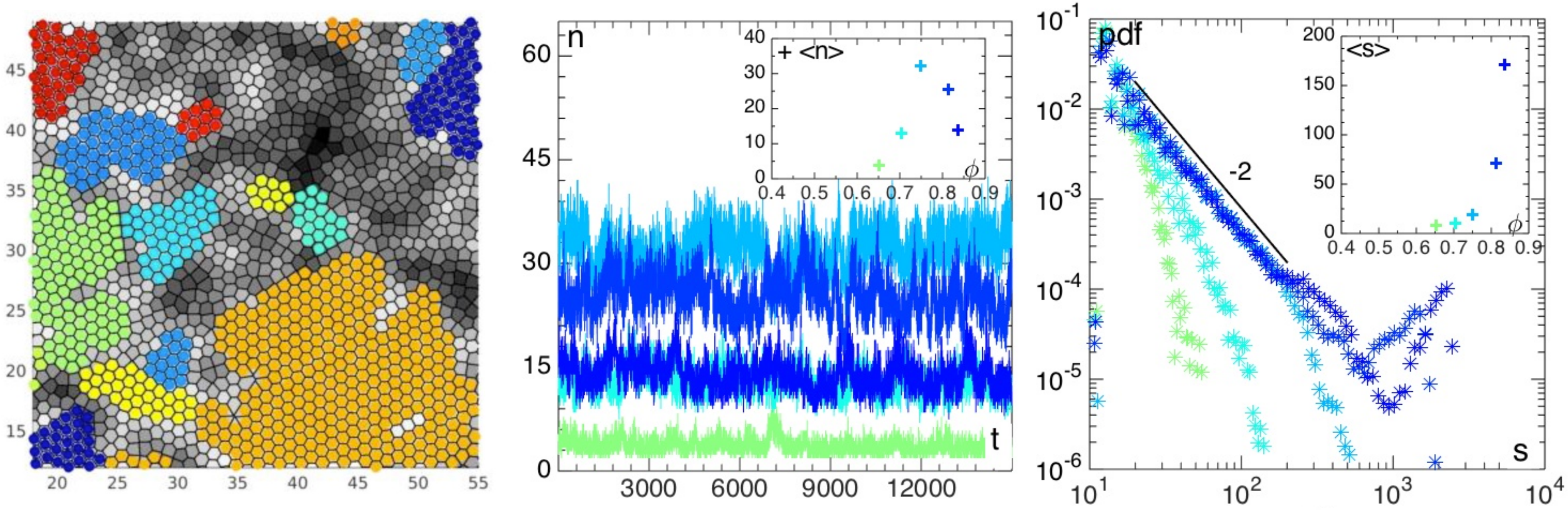
Dynamics : structural relaxation

$$Q(a,\tau) = \left\langle \frac{1}{N} \sum_p \exp - \frac{[\mathbf{r}_p(t+\tau) - \mathbf{r}_p(t)]^2}{a^2} \right\rangle$$



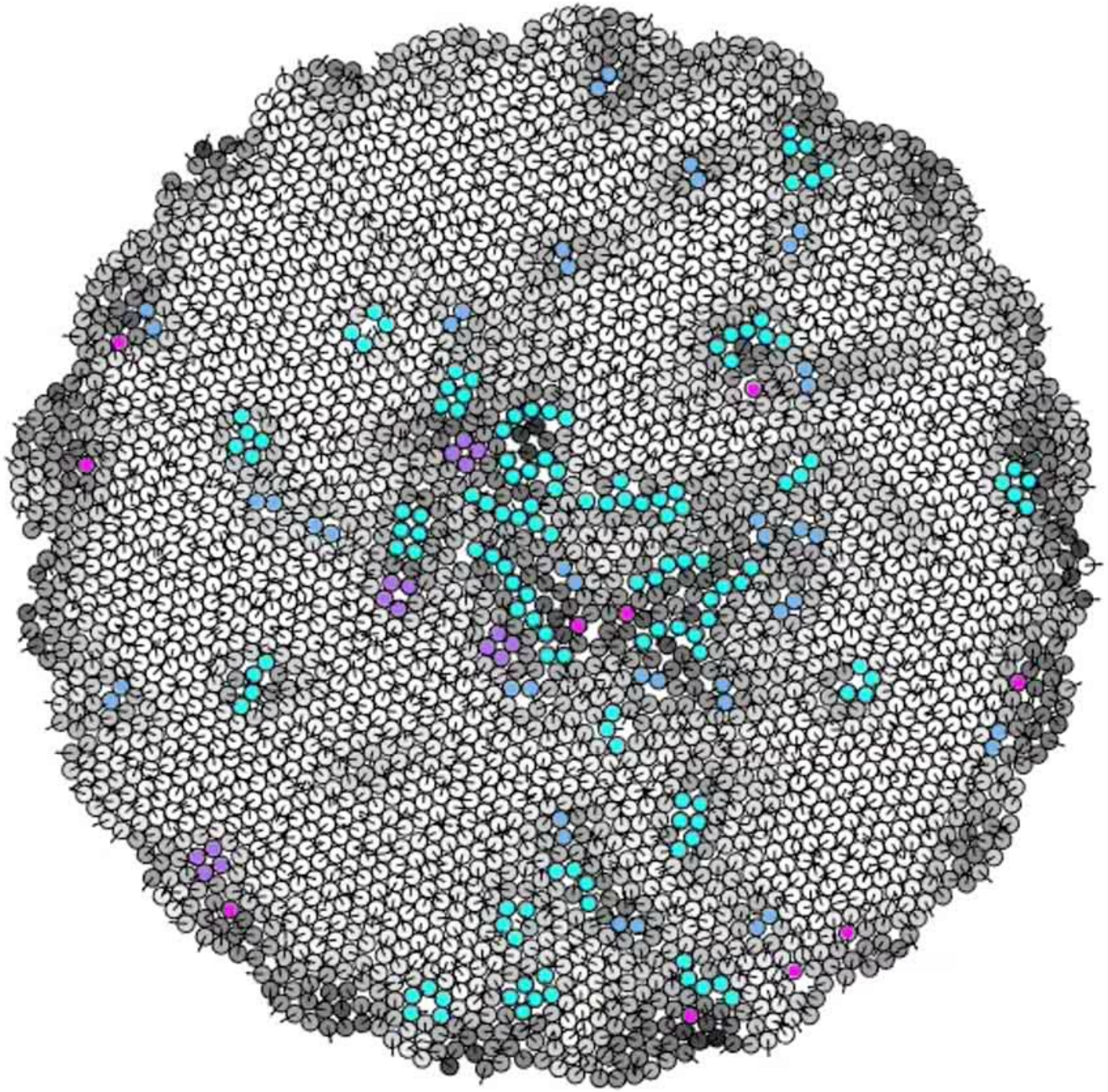
- Indeed a rather weak slowing down of the dynamics …
- The whole structure relaxes => a very different image from phase coexistence

A liquid of clusters

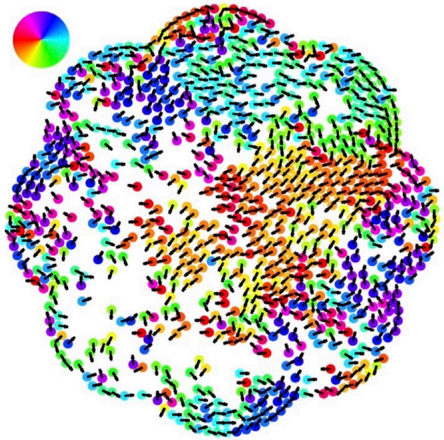


- No coarsening : a steady number and distribution of cluster
- An increasing number of fluctuating clusters
- A “percolation” like transition towards a system size dominating cluster

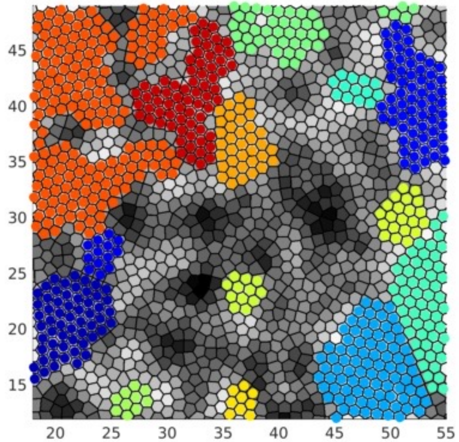
Proliferation of highly motile defects



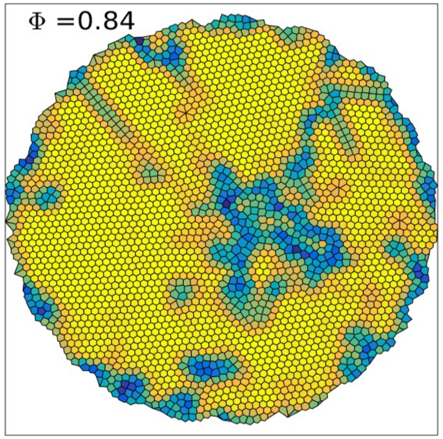
A first draft for a phase diagram



Active liquid

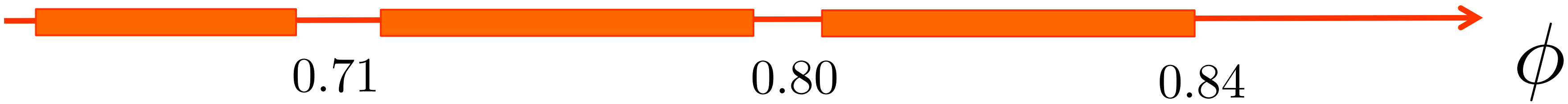


Liquid of clusters



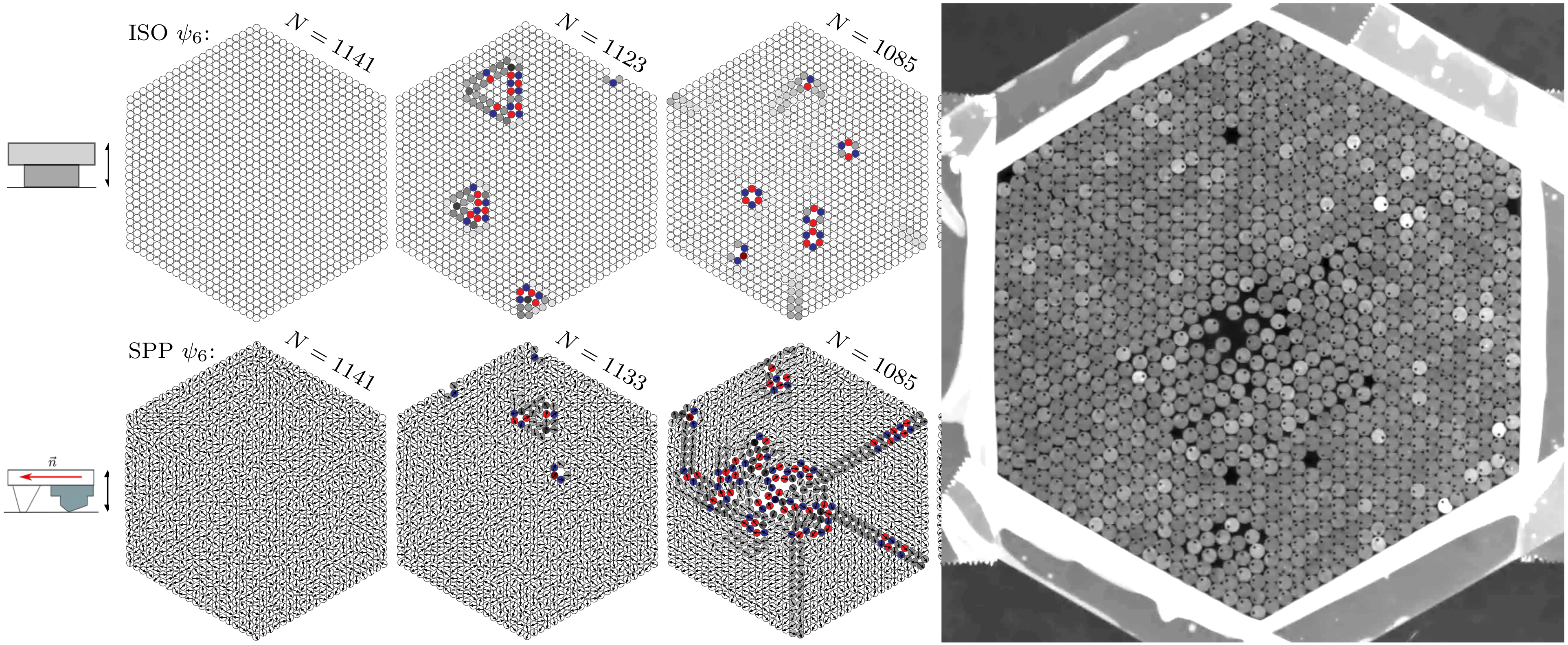
Percolating Cluster
+ Fragmentation

?

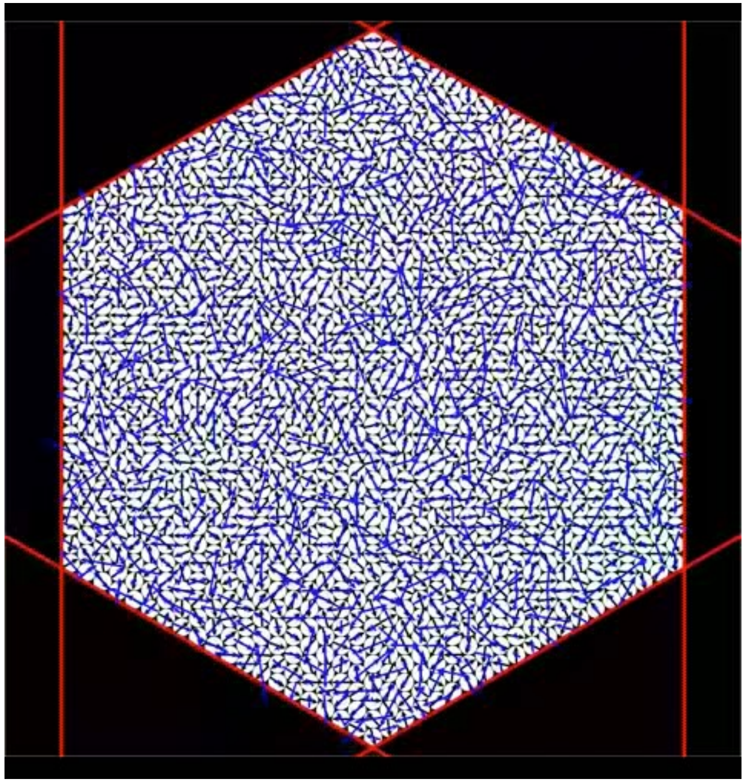


- What about higher packing fraction ?
- What if the boundaries do not frustrate the hexagonal symmetry ?

Active crystal of hard discs close to Ordered Closed Packing



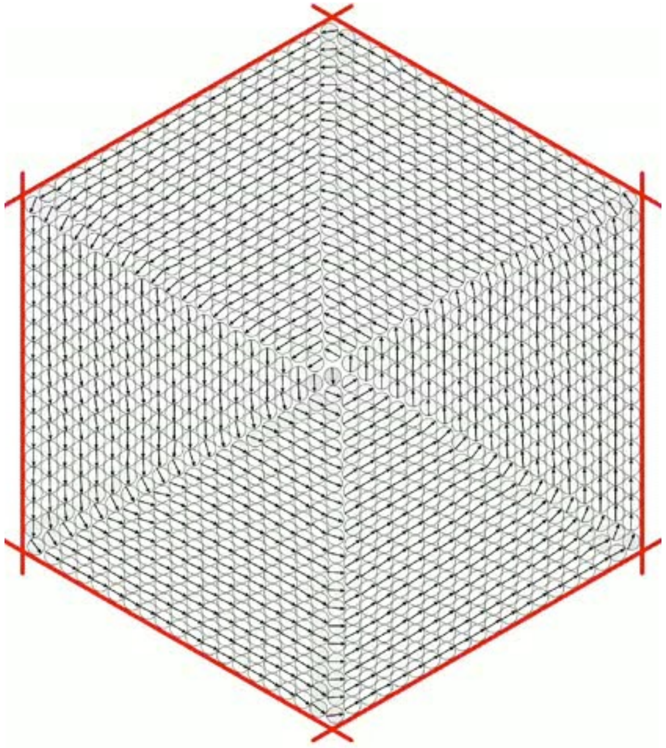
Numerics



Experimental conditions

$$\tau_v \dot{\mathbf{v}} = \hat{\mathbf{n}} - \mathbf{v} + F_{int}$$

$$\tau_n \dot{\hat{\mathbf{n}}} = (\hat{\mathbf{n}} \times \mathbf{v}) \times \mathbf{n} + \sqrt{2D}\xi \mathbf{n}_\perp$$

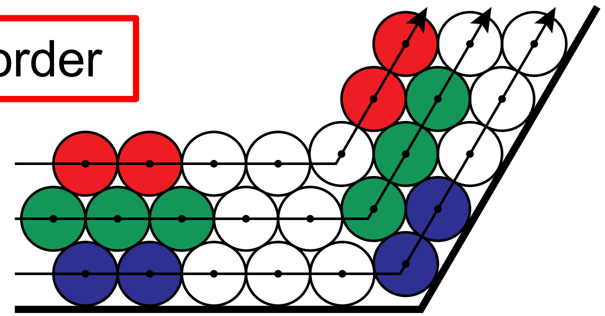


In the noiseless limit

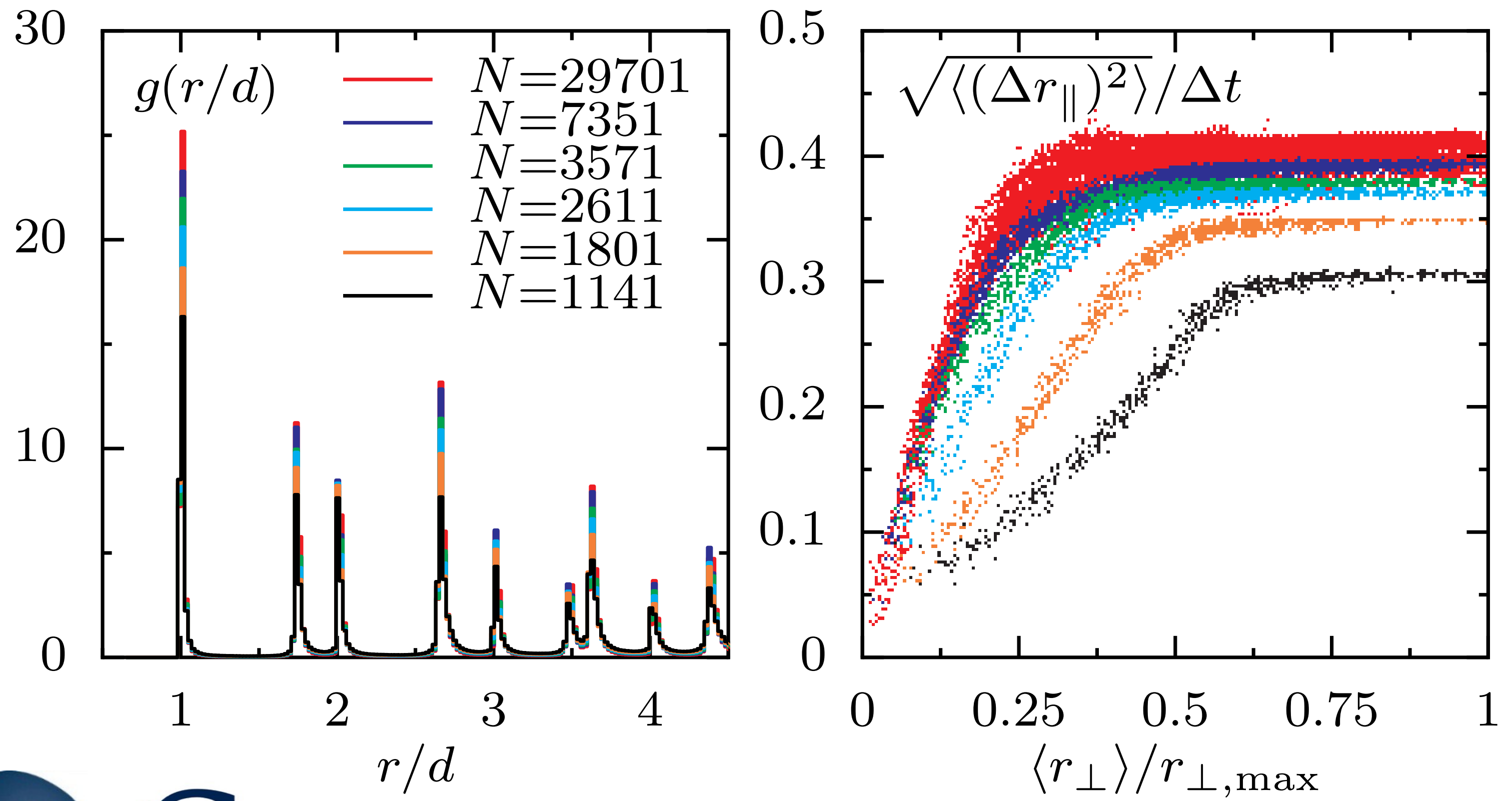
A bona fide flowing crystaline phase !

Structure and dynamics within the hexagon

Shear localizes on stacking faults to preserve structural order



The larger, the more ordered, the faster



Outline: from active liquids to active solids

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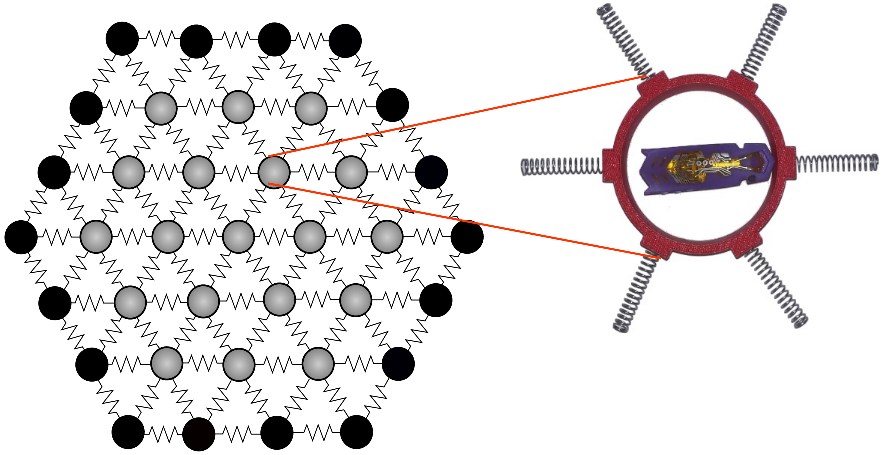
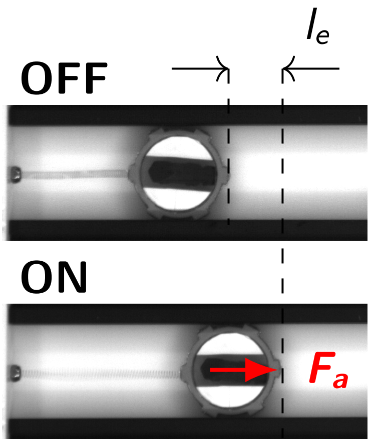
  - mechanical pressure is not a state variable
  - liquid-gas phase separation takes place in purely repulsive systems
  - macroscopic flows emerge in the absence of external gradient

- Active solids :
  - spontaneous flows also take place in crystalline structure
  - selective & collective actuation emerges in linear elastic systems

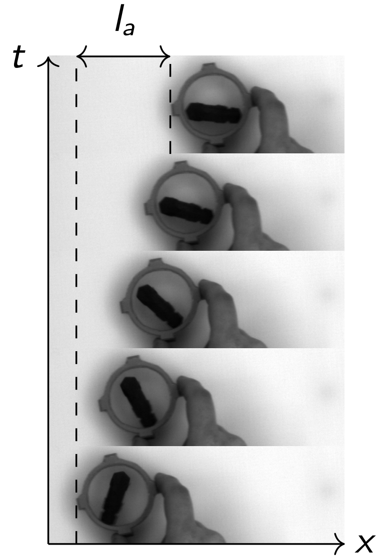
Active elastic lattices : the epitome of active solids

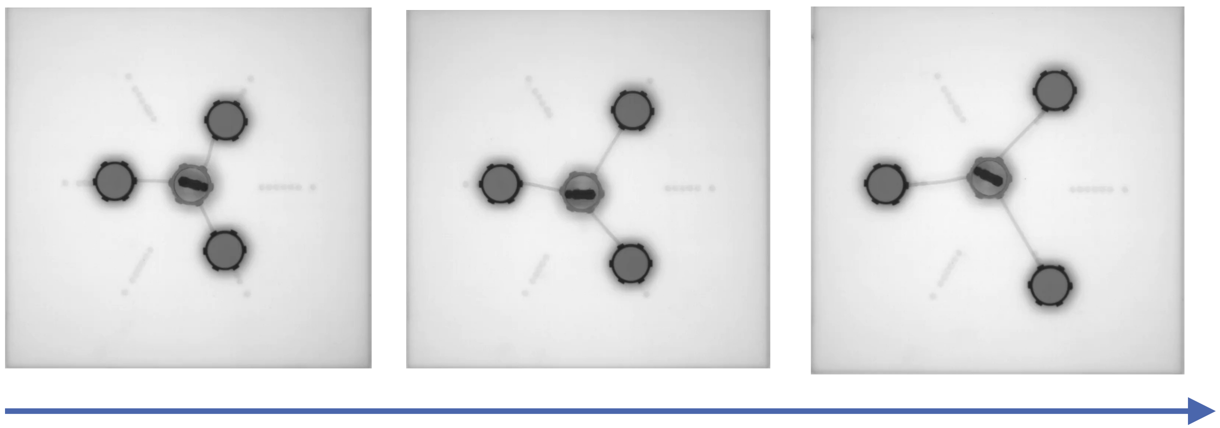


- Linear elasticity + active force



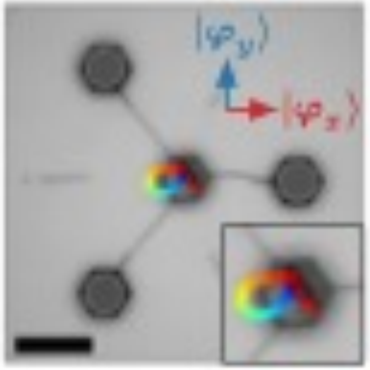
- Self alignment





$$\pi = \frac{l_e}{l_a} = \frac{F_0}{k l_a}$$

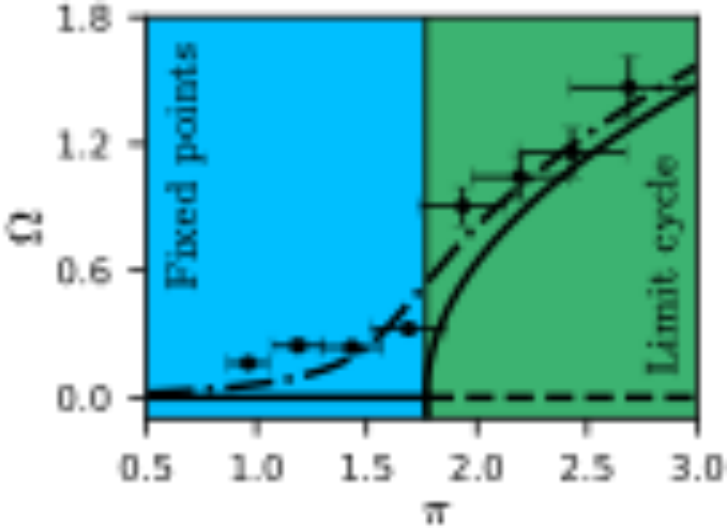
The one particle problem

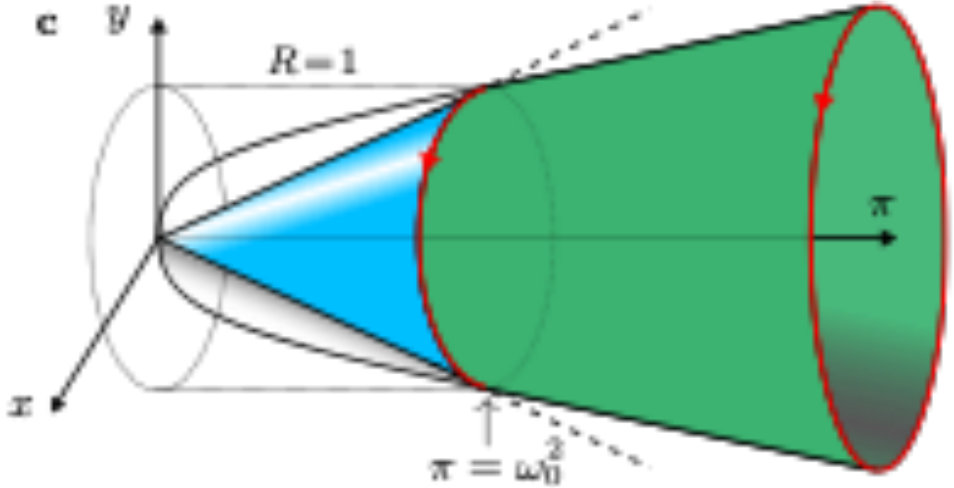


$$|\dot{\boldsymbol{u}}\rangle = \pi|\hat{\mathbf{n}}\rangle - \mathbb{M}|\boldsymbol{u}\rangle$$
$$|\dot{\boldsymbol{n}}\rangle = -\mathbb{K}^T\mathbb{K}\mathbb{M}|\boldsymbol{u}\rangle$$

An infinite set of fixed points

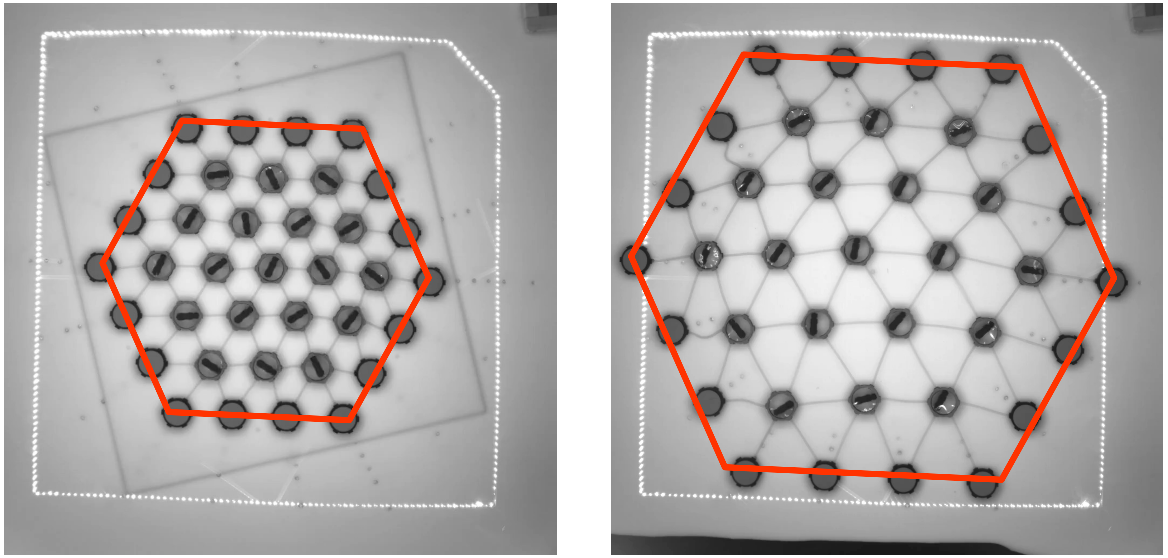
$$\left\{|\mathbf{u}\rangle = \pi\mathbb{M}^{-1}|\hat{\mathbf{n}}\rangle, |\hat{\mathbf{n}}\rangle\right\}$$





From collective motion to collective actuation

Pinned boundary conditions
=> No zero mode



$$\pi = \frac{l_e}{l_a} = \frac{F_0}{k l_a}$$

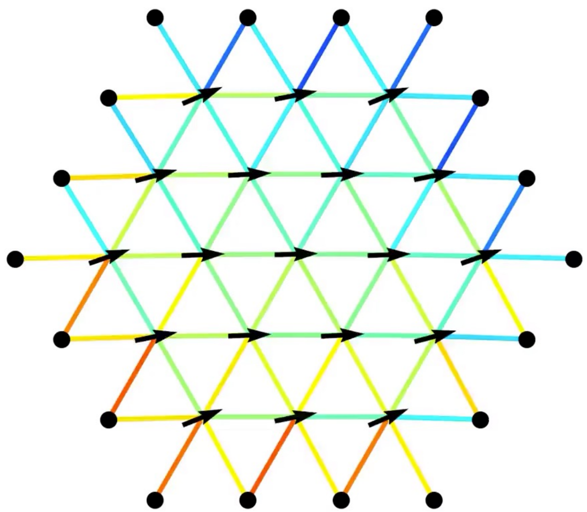
Collective actuation in overdamped and harmonic dynamics

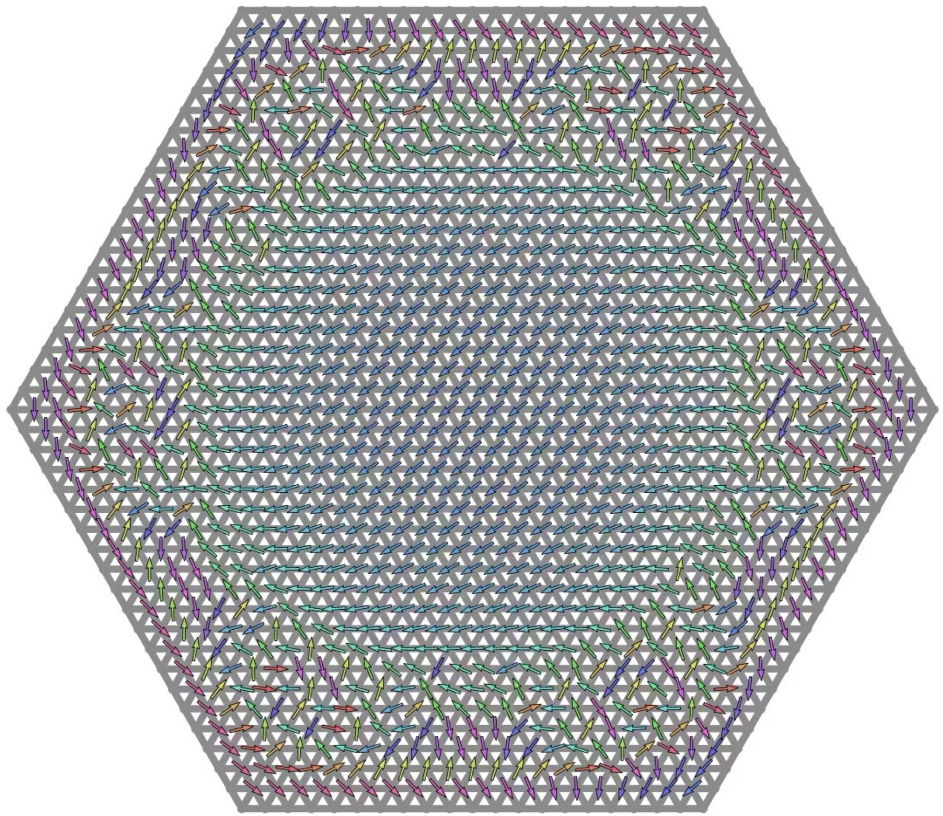
$$\tau_v \dot{\boldsymbol{v}}_i = \hat{\boldsymbol{n}}_{\boldsymbol{i}} - \boldsymbol{v}_i + \boldsymbol{F}_i$$
$$\tau_n \dot{\boldsymbol{n}}_i = (\hat{\boldsymbol{n}}_{\boldsymbol{i}} \times \boldsymbol{v}_i) \times \hat{\boldsymbol{n}}_{\boldsymbol{i}}$$

overdamped
+ harmonic

=>

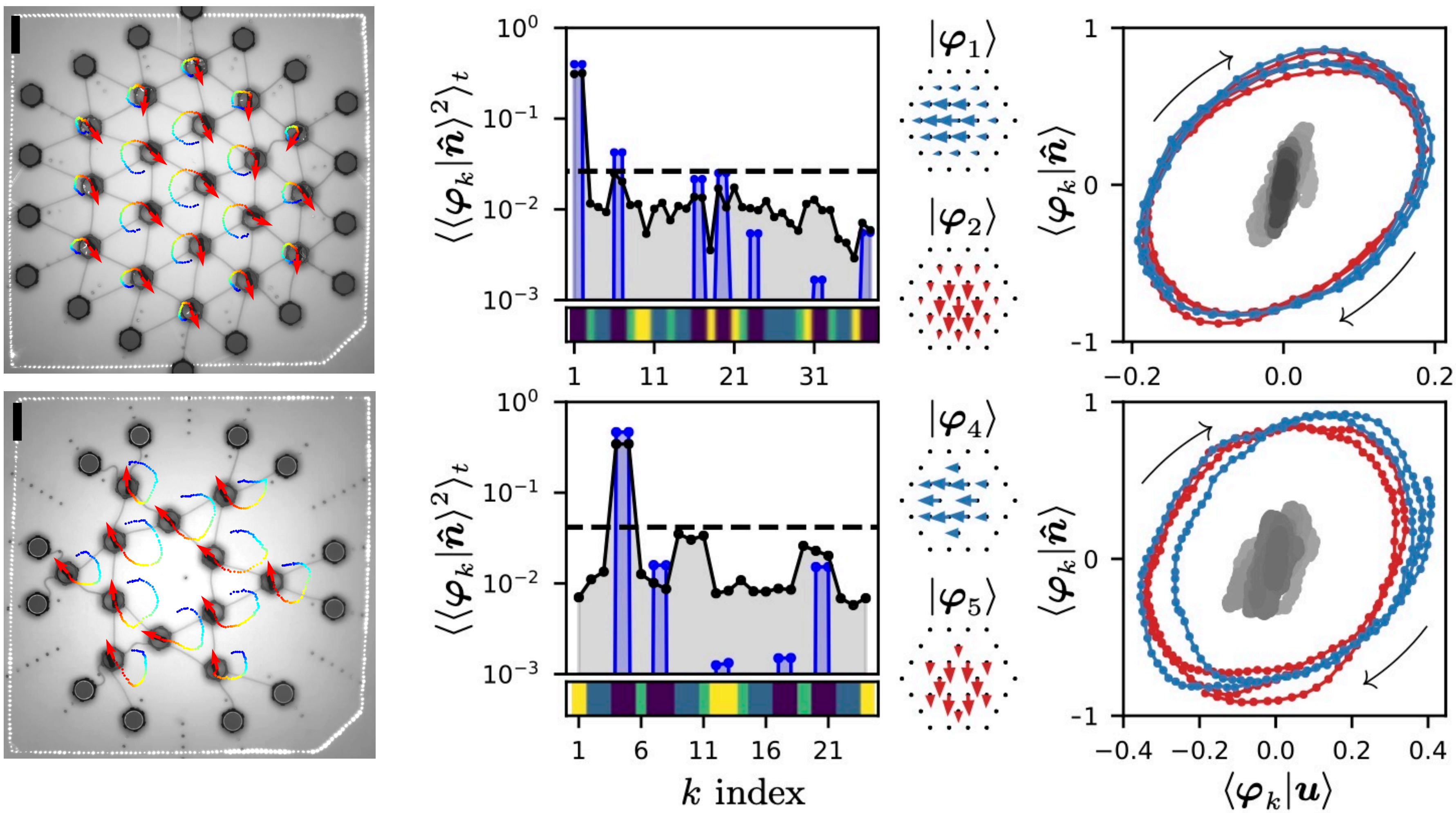
$$\dot{\boldsymbol{u}}_i = \pi \hat{\boldsymbol{n}}_{\boldsymbol{i}} - \mathbb{M}_{ij} \boldsymbol{u}_{\boldsymbol{j}}$$
$$\dot{\boldsymbol{n}}_i = -(\hat{\boldsymbol{n}}_{\boldsymbol{i}} \times \mathbb{M}_{ij} \boldsymbol{u}_{\boldsymbol{j}}) \times \hat{\boldsymbol{n}}_{\boldsymbol{i}}$$





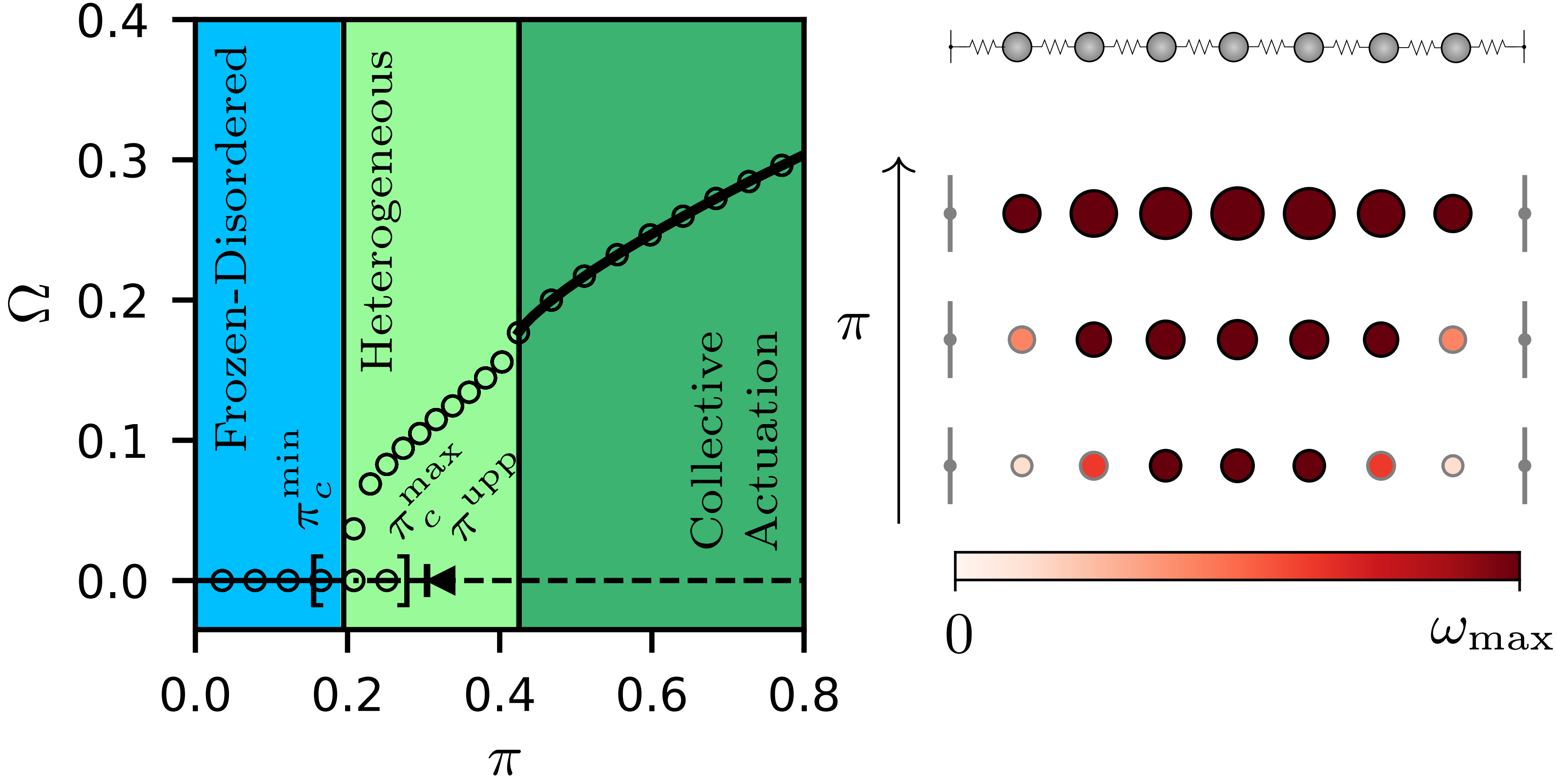
A solid dynamical chiral phase with spontaneously broken parity symmetry

Collective actuation takes place on a few selected modes

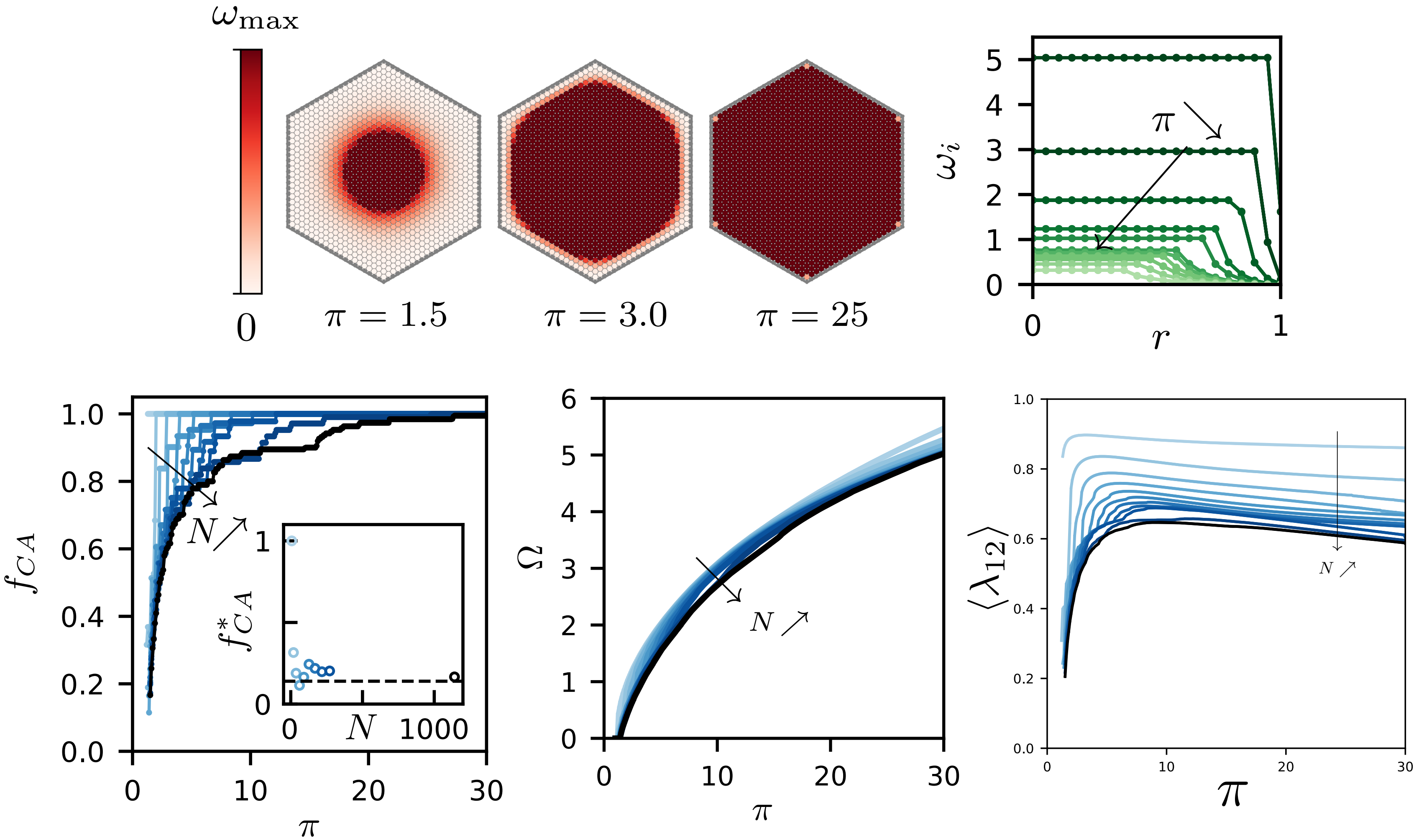


A non trivial modal selection, rooted in the mode geometries

N particles in a chain



The transition to collective actuation is discontinuous



Coexistence between the frozen disordered phase and the chiral one

Outlook

- Active matter physics started with the study of collective motion in flocks of birds in 1995.
- In the past 25 years, active liquids have driven a very intense research
  - physicists have designed a large amount of model experimental systems and numerical models
  - => the observations of a bunch of striking and interesting phenomena
  - kinetic and field theories => a rather good understanding of these phenomena
- More recently the study of biological tissues has driven the attention towards highly dense systems, eventually behaving as solids rather than liquids
  - A lot remains to be done to fully understand the physics of active solids.
  - Tools of (harder) condensed matter physics are likely to become increasingly helpful

Mechanical Pressure :	*Phys. Rev. Lett.* **119** 028002 (2017).
Collective motion of discs :	*Phys. Rev. Lett.* **105,** 098001 (2010)
	Phys. Rev. Lett. **110,** 208001 (2013).
Collective motion of colloids :	*Nature* **503,** 95–98 (2013).
Flowing Crystal of discs :	*Phys. Rev. Lett.* **120,** 208001 (2018).
Collective actuation :	In preparation

THANK YOU!